GEOS F493 / F693
Geodetic Methods and Modeling

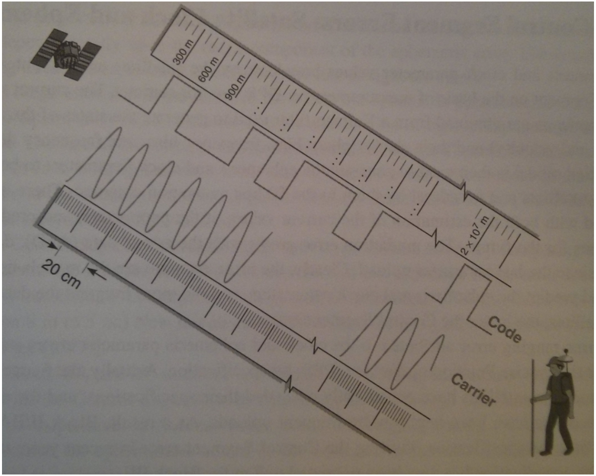
– Lecture 04a: GPS Carrier Phase –

Ronni Grapenthin
rgrapenthin@alaska.edu
Elvey 413B
(907) 474-7286

September 23, 2019

Measurement Models

- Code Phase Measurement (last week)
- Carrier Phase Measurement (today!)



Misra and Enge, 2011, GPS–Signals, Measurements, and Performance

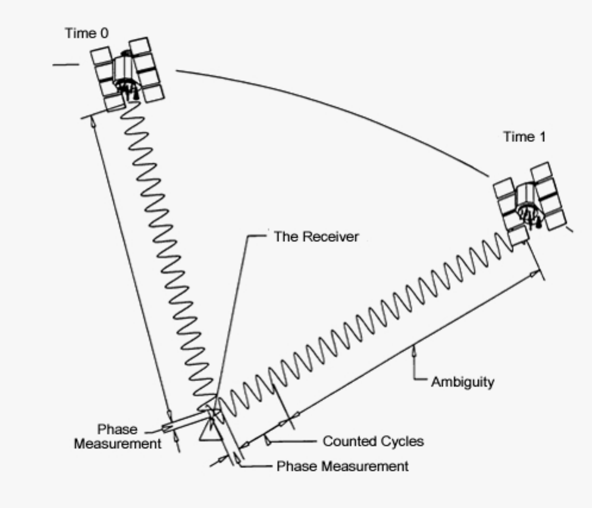
Carrier Phase Measurement

- also: carrier beat phase measurement
- difference between phases of receiver generated carrier signal and carrier received from satellite
- is indirect and ambiguous measurement of signal transit time
- phase at time *t*:

$$\phi(t) = \phi_u(t) - \phi^s(t - \tau) + N$$

-
    - $\phi_u(t)$ phase of rcx generated signal
    - $\phi^S(t - \tau)$ phase of satellite signal received at t (sent at $t - \tau$)
    - τ: still transit time
    - N: integer ambiguity, must be estimated: *integer ambiguity resolution*

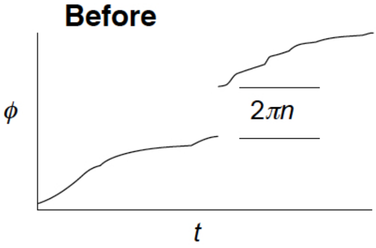
Phase (Integer) Ambiguity

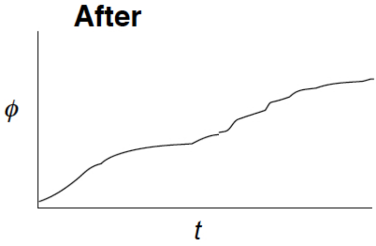


http://nptel.ac.in/courses/105104100/lectureB_8/B_8_4carrier.htm

Cycle Slip

- receiver has to track phase continuously
- loss of lock (tree, etc): cycle slip – integer number of cycles jump in phase data
- must be fixed during analysis (software, several strategies; sometimes manually)





courtesy: *Jeff Freymueller*

Carrier Phase Measurement

$$\phi = \frac{1}{\lambda} * (r + I + T) + f * (\delta t_u - \delta t^s) + N + \epsilon_\phi$$

(units of cycles) where

- λ, f - carrier wavelength, frequency
- r - geometric range
- I, T - ionospheric, tropospheric propagation errors (path delays)
- $\delta t_u, \delta t^s$ - receiver, satellite clock biases
- N - phase ambiguity
- ϵ_ϕ - error term (phase)

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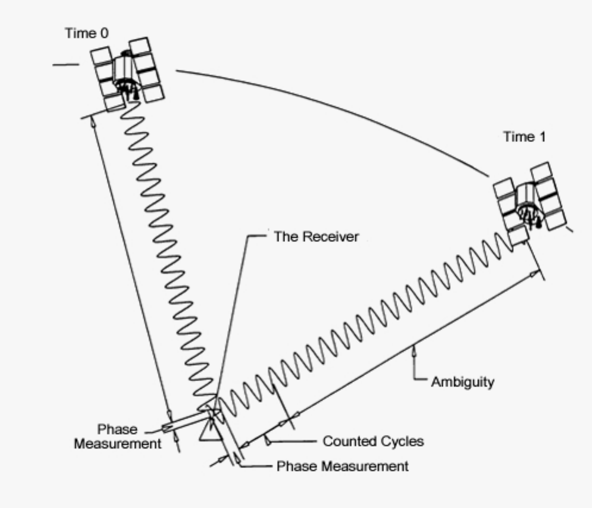
compare to code measurement eqn (units of distance):

$$\rho = r + I + T + c * (\delta t_u - \delta t^s) + \epsilon_\rho$$

Code tracking is unambiguous (because codes are long!)
$\sigma(\epsilon_\rho) \approx 0.5\,\mathrm{m}$
$\sigma(\epsilon_\phi) \approx 0.025$ cycle (5 mm)

Phase Ambiguity



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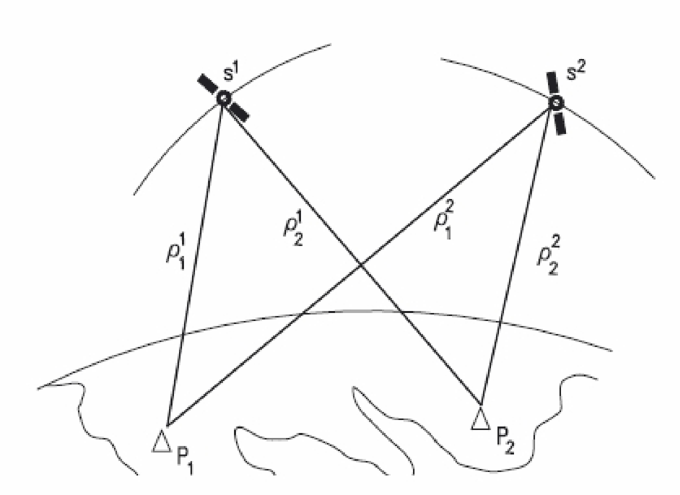
Elimination of “Nuisance” Parameters

- difference multiple satellite and receiver data to eliminate clock biases
- “single difference” between 2 receivers and 1 satellite: eliminates satellite clock
- “single difference” between 1 receiver and 2 satellites: eliminates receiver clock
- “double difference” between those differences removes both clocks
- BUT: you estimate baseline vector between receivers rather than their positions!
- no linearly dependent observations, careful choosing (by software)
- some estimate clock errors intead

Single + Double Difference



http://www.fig.net/resources/publications/figpub/pub49/figpub49.asp

Single Difference

Carrier phase measurement from satellite k at receiver u:

$$\phi_u^{(k)} = \frac{1}{\lambda} * (r_u^{(k)} + I_u^{(k)} + T_u^{(k)}) + f * (\delta t_u - \delta \mathbf{t}^{(\mathbf{k})}) + N_u^{(k)} + \epsilon_{\phi,u}^{(k)}$$

Single Difference

Carrier phase measurement from satellite *k* at receiver *u*:

$$\phi_u^{(k)} = \frac{1}{\lambda} * (r_u^{(k)} + I_u^{(k)} + T_u^{(k)}) + f * (\delta t_u - \delta \mathbf{t}^{(\mathbf{k})}) + N_u^{(k)} + \epsilon_{\phi,u}^{(k)}$$

Carrier phase measurement from satellite *k* at receiver *r*:

$$\phi_r^{(k)} = \frac{1}{\lambda} * (r_r^{(k)} + I_r^{(k)} + T_r^{(k)}) + f * (\delta t_r - \delta \mathbf{t}^{(\mathbf{k})}) + N_r^{(k)} + \epsilon_{\phi,r}^{(k)}$$

Single Difference

Carrier phase measurement from satellite k at receiver u:

$$\phi_u^{(k)} = \frac{1}{\lambda} * (r_u^{(k)} + I_u^{(k)} + T_u^{(k)}) + f * (\delta t_u - \delta \mathbf{t}^{(\mathbf{k})}) + N_u^{(k)} + \epsilon_{\phi,u}^{(k)}$$

Carrier phase measurement from satellite k at receiver r:

$$\phi_r^{(k)} = \frac{1}{\lambda} * (r_r^{(k)} + I_r^{(k)} + T_r^{(k)}) + f * (\delta t_r - \delta \mathbf{t}^{(\mathbf{k})}) + N_r^{(k)} + \epsilon_{\phi,r}^{(k)}$$

receiver single difference:

$$\begin{aligned} \phi_{ur}^{(k)} &= \phi_u^{(k)} - \phi_r^{(k)} \\ &= \frac{1}{\lambda} * (r_{ur}^{(k)} + I_{ur}^{(k)} + T_{ur}^{(k)}) + f * \delta t_{ur} + N_{ur}^{(k)} + \epsilon_{\phi,ur}^{(k)} \end{aligned}$$

Single Difference

Carrier phase measurement from satellite k at receiver u:

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receiver single difference:

$$\begin{aligned}\phi_{ur}^{(k)} &= \phi_u^{(k)} - \phi_r^{(k)} \\ &= \frac{1}{\lambda} * (r_{ur}^{(k)} + I_{ur}^{(k)} + T_{ur}^{(k)}) + f * \delta t_{ur} + N_{ur}^{(k)} + \epsilon_{\phi,ur}^{(k)}\end{aligned}$$

"short" baseline (ionosphere, troposphere errors small)

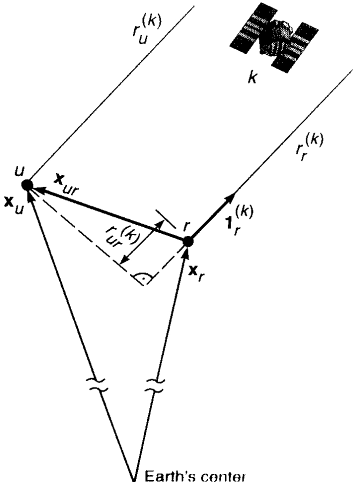
$$\phi_{ur}^{(k)} = \frac{r_{ur}^{(k)}}{\lambda} + f * \delta t_{ur} + N_{ur}^{(k)} + \epsilon_{\phi,ur}^{(k)}$$

Single Difference

want to estimate $\mathbf{x_{ur}} = \mathbf{x_u} - \mathbf{x_r}$ hidden in range difference (short baselines):

$$r_{ur}^{(k)} = r_u^{(k)} - r_r^{(k)} = -\mathbf{1_r^{(k)}} \mathbf{x_{ur}}$$

$\mathbf{1_r^{(k)}}$ is unit vector pointing from receiver *r* to satellite *k* (different treatment for longer baselines)



Misra and Enge, 2011, GPS–Signals, Measurements, and Performance

Double Difference

Form single differences for receivers u, r and satellite l

$$
\begin{aligned}
\phi_{ur}^{(l)} &= \phi_{u}^{(l)} - \phi_{r}^{(l)} \\
&= \frac{r_{ur}^{(l)}}{\lambda} + \mathbf{f} * \delta \mathbf{t_{ur}} + N_{ur}^{(l)} + \epsilon_{\phi,ur}^{(l)}
\end{aligned}
$$

Double Difference

Form single differences for receivers u, r and satellite l

$$\begin{aligned} \phi_{ur}^{(l)} &= \phi_u^{(l)} - \phi_r^{(l)} \\ &= \frac{r_{ur}^{(l)}}{\lambda} + \mathbf{f} * \delta \mathbf{t_{ur}} + N_{ur}^{(l)} + \epsilon_{\phi,ur}^{(l)} \end{aligned}$$

Form double difference:

$$\begin{aligned} \phi_{ur}^{(kl)} &= \phi_{ur}^{(k)} - \phi_{ur}^{(l)} \\ &= (\phi_u^{(k)} - \phi_r^{(k)}) - (\phi_u^{(l)} - \phi_r^{(l)}) \\ &= \frac{r_{ur}^{(kl)}}{\lambda} + N_{ur}^{(kl)} + \epsilon_{\phi,ur}^{(kl)} \end{aligned}$$

Triple Difference

Triple Difference

- adds difference in time
- difference double difference from epoch t_1 and t_0
- can be used to eliminate phase ambiguity
- but removes most of geometric strength and hence gives weak positions

Geometry Issues

- Position estimate depends on quality (ϵ) and geometry (θ) of range measurement

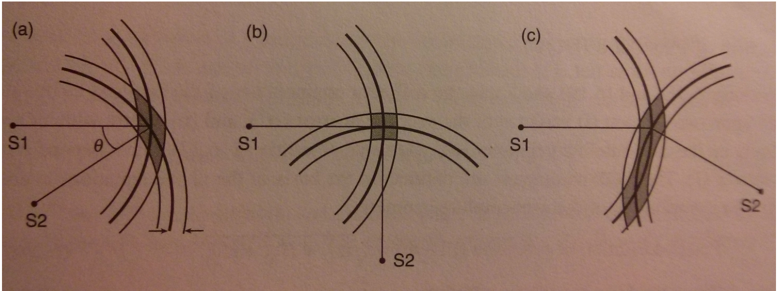


Figure 6.2 A simple 2-D example of position estimation with range measurements. The quality of the position estimate depends upon both the quality (ε) and geometry (θ) of the range measurements. The shaded region represents the uncertainty in the position estimate.

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Carrier Phase Measurement

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(units of cycles) where

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compare to code measurement eqn (units of distance):

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Code tracking: unambiguous (long!)
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$\sigma(\epsilon_\phi) \approx 0.025\,\text{cycle}$ (5 mm)

Integer Ambiguity Resolution 1/5

- uncertainty in integer estimation depends on carrier wavelength
- increase wavelength –> decrease uncertainty: create *wide lane measurement*:

$$\begin{aligned}\phi_{L12} &= \phi_{L1} - \phi_{L2} \\ &= r(f_{L1} - f_{L2})/c + (N_{L1} - N_{L2}) + \epsilon_{\phi_{L12}} \\ &= r/\lambda_{L12} + N_{L12} + \epsilon_{\phi_{L12}}\end{aligned}$$

where $\lambda_{L12} = c/(f_{L1} - f_{L2}) = 0.862\,\text{m}$
$f_{L12} = f_{L1} - f_{L2} = 347.82\,\text{MHz}$
N_{L12} is integer ambiguity

Integer Ambiguity Resolution 2/5

Using

$$\rho_{L1} = r + \epsilon_{\rho_{L1}}$$

we can form estimate of N_{L12} as:

$$N_{L12} \approx \left[\phi_{L12} - \frac{\rho_{L1}}{\lambda_{L12}} \right]_{roundoff}$$

Here, $\sigma(N_{L12}) \approx 1.2$ cycles;
compared to $\sigma(N_{L1}) \approx 5$ cycles

Integer Ambiguity Resolution 3/5

- wide lane measurements much noisier than L1,L2 measurements
- *narrow lane combination* $\phi_{Ln} = \phi_{L1} + \phi_{L2}$ less noisy
- though harder to resolve ambiguities with narrow lane
- position estimates would be more precise

Integer Ambiguity Resolution (one at a time) 4/5

With correct N_{L12} can determine N_{L1}, N_{L2}. Measurement eqs:

$$\begin{aligned}\phi_{L1} &= r/\lambda_{L1} + N_{L1} + \epsilon_{\phi_{L1}} \\ \phi_{L2} &= r/\lambda_{L2} + N_{L2} + \epsilon_{\phi_{L2}}\end{aligned}$$

after solving both for r and equating, we get:

$$N_{L1} - \frac{\lambda_{L2}}{\lambda_{L1}} N_{L2} = \phi_{L1} - \frac{\lambda_{L2}}{\lambda_{L1}} \phi_{L2} + \epsilon$$

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$$\begin{aligned} N_{L1} - N_{L2} &= N_{L12} \\ N_{L2} &= N_{L1} - N_{L12} \end{aligned}$$

Integer Ambiguity Resolution (one at a time) 4/5

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So, we can solve for N_{L1}, N_{L2}:

$$N_{L1} = \left[\frac{\lambda_{L2}}{\lambda_{L1}} - 1\right]^{-1} \left[\frac{\lambda_{L2}}{\lambda_{L1}} N_{L12} - \phi_{L1} + \frac{\lambda_{L2}}{\lambda_{L1}} \phi_{L2}\right]$$

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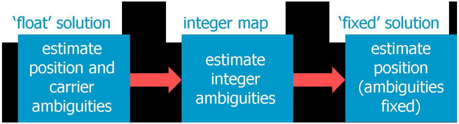
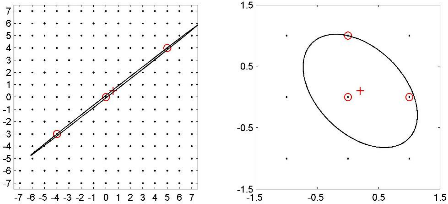
Uncertainty $\sigma(N_{L1}) \approx 6\sigma(\epsilon_{\phi_{L_1}})$; data quality determines success.

Integer Ambiguity Resolution (as a set) 5/5

- 1) discard integer nature of ambiguities and find least squares 'float solution'
- 2) map to integer (decorrelate error elipse)
- 3) 'fixed solution': estimate position (other parameters) w/ integer ambiguities





http://www.citg.tudelft.nl/en/about-faculty/departments/geoscience-and-remote-sensing/research-themes/gps/lambda-method/