GEOS F493 / F693
Geodetic Methods and Modeling

– Lecture 04b: GPS Error Sources / Models –

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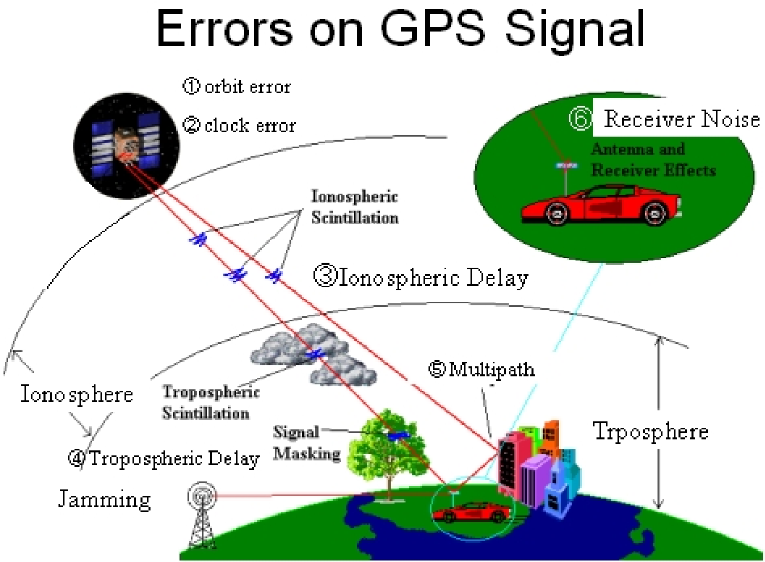
GPS Noise Sources



http://www.blackboxcamera.com

Ionospheric Delay 1/5

carrier phase (unit of cycles):

$$\phi = \frac{1}{\lambda} * (r + \mathbf{I} + T) + f * (\delta t_u - \delta t^s) + N + \epsilon_\phi$$

code measurement eqn (units of distance):

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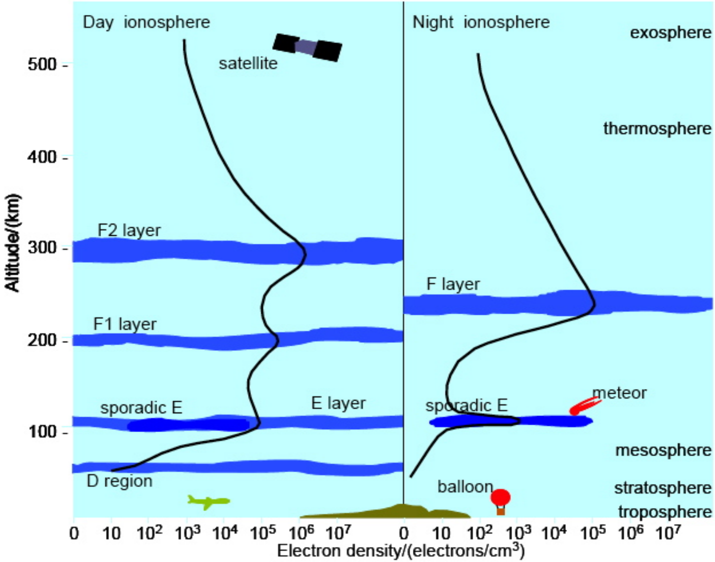
Ionospheric Delay 2/5

- $\approx$ 50-1000 km above Earth
- ionized gases: free electrons and ions, sun's radiation/activity drives state
- daily cycle with peak electron density at about 2 pm local time
- electron density 1-2 orders of magnitude difference between night/day
- changes w/ seasons, 11-year solar cycle, other short term anomalies (tsunamis), solar flares
- dispersive for GPS frequencies (different frequencies - different effective velocity)

Ionospheric Delay 3/5



http://www.amateur-radio-wiki.net

Ionospheric Delay 4/5

Total electron content (TEC): number of electrons $n_e(l)$ in tube of 1 m^2 connecting satellite and receiver:

$$TEC = \int_R^S n_e(l) dl \text{ (TECU: TEC units)}$$

- *VTEC*: TEC in vertical direction, lowest TEC when satellite in zenith direction
- *VTEC* between 1-150 TECU
- region with highest ionospheric delay within $\pm 20^\circ$ of magnetic equator

Ionospheric Delay 5/5

Dual-frequency receivers allow (basically) elimination of ionosphere as source of error. Possible combinations for code range and carrier phases (derivation in Hofmann-Wellenhof et al.)

$$\begin{aligned} \rho_{L1,L2} &= \rho_{L1} - \frac{f_{L2}^2}{f_{L1}^2}\rho_{L2} \\ \phi_{L1,L2} &= \phi_{L1} - \frac{f_{L2}}{f_{L1}}\phi_{L2} \end{aligned}$$

other combinations possible.

Model exists to remove ionosphere from L1-only observations (*Klobuchar model*, [Klobuchar, 1996]).

Tropospheric Delay 1/4

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- refractivity of air parcel depends on temperature, partial pressures of dry gases, water vapor
- approximations: $N_d = 77.64P/T$ and $N_w = 3.73 \times 10^5 e/T^2$ with P total, e partial pressures (mB), T temperature in K.

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- models: differ in assumptions regarding changes of temperature and water vapor with altitude
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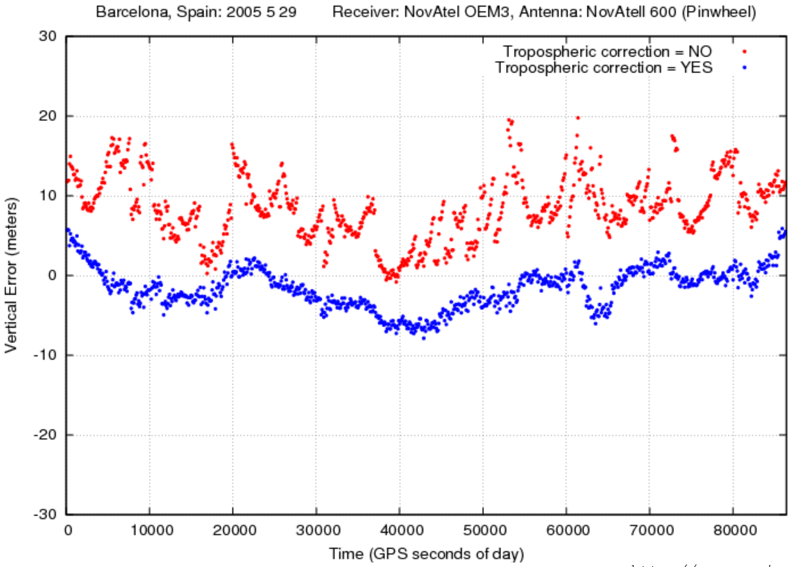
- lots of both exist

Tropospheric Delay 4/4



http://www.navipedia.net

Receiver Noise

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Receiver Noise

- RF radiation sensed by antenna (interference)
- noise introduced by antenna, amplifiers, cables (!), receiver, signal quantization!
- absence of interference: rcx sees waveform = GPS + random noise
- fine structure of signal can be masked by noise (esp. low SNR)
- error varies w/ signal strength, which depends on satellite elev angle

Multipath

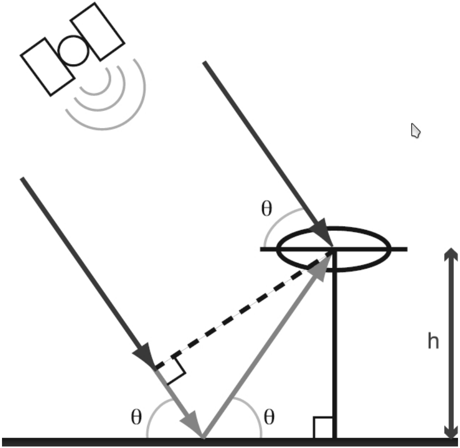
carrier phase (unit of cycles):

$$\phi = \frac{1}{\lambda} * (r + I + T) + f * (\delta t_u - \delta t^s) + N + \mathbf{MP} + \epsilon_\phi$$

code measurement eqn (units of distance):

$$\rho = r + I + T + c * (\delta t_u - \delta t^s) + \mathbf{MP} + \epsilon_\rho$$

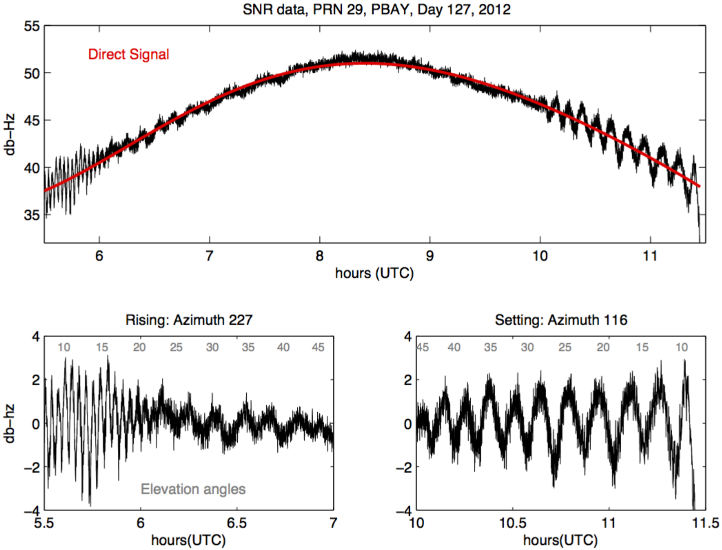
Multipath



Larson et al. (2007)

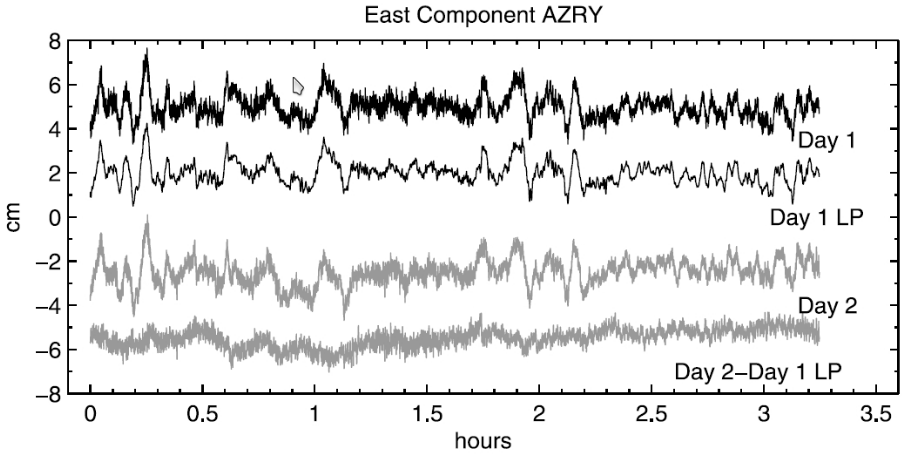
- best seen in subdaily solutions
- signal reaches antenna via direct and indirect paths
- reflected signal delayed, weaker
- mitigation: antenna design, receiver algorithms
- code and phase measurement are sum of received signals
- pseudorange: 1-5 m error
- phase: 1-5 cm error (no worse than 1/4 cycle)

Multipath in SNR data



http://xenon.colorado.edu/spotlight/index.php?product=spotlight&station=PBAY

Eliminating Multi-Path through Sidereal Filtering



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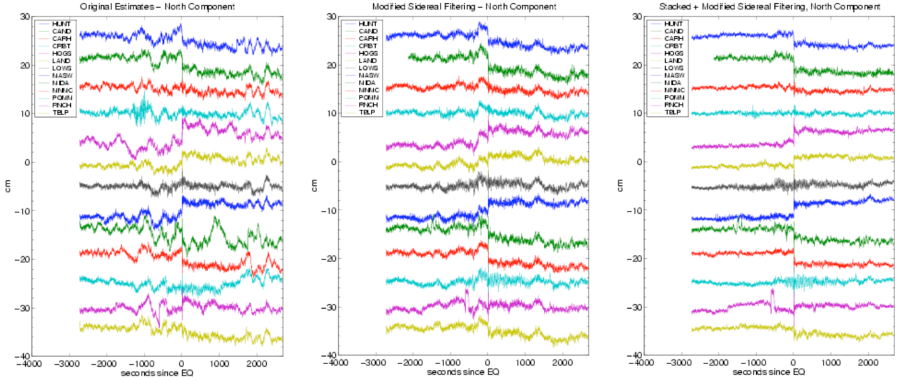
Eliminating Multi-Path through Sidereal Filtering

Parkfield earthquake



Andria Bilich, University of Colorado

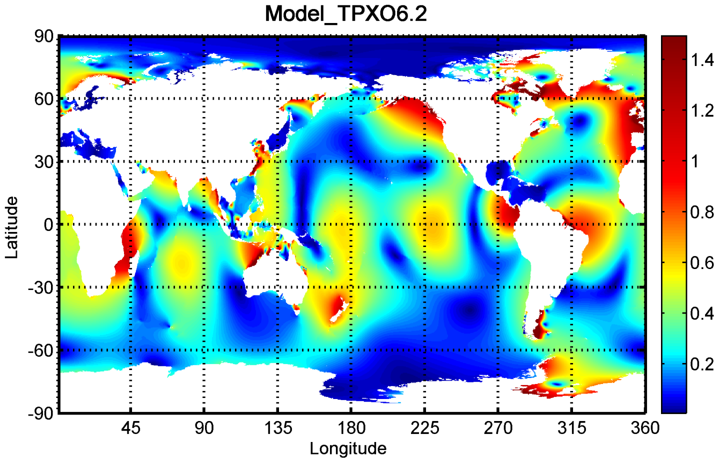
Ocean Tidal Loading

- solid earth responds to changing load due to ocean tides
- large near coast (with large tidal range, depends on coastline)
- need good tidal models for removal

e.g., TPXO6:

- eight primary constituents M2, S2, N2, K2, K1, O1, P1, Q1
- two long period Mf,Mm constituents
- three non-linear M4, MS4, MN4 harmonic constituents
- on 1/4 degree resolution full global grid (for versions 6.* and later).

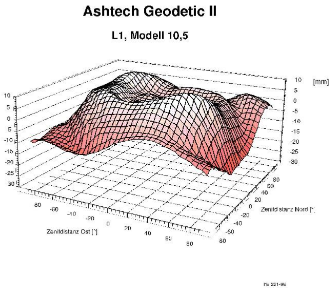
Ocean Tidal Loading

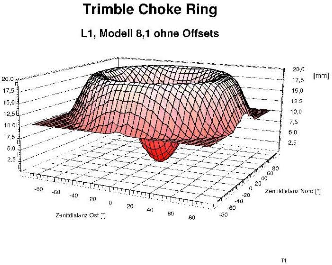


Map of M2 sea surface height amplitude (m) from TPXO6.2

https://www.esr.org/polar_tide_models/Model_TPXO62.html

Antenna Phase Center Models





- imaginary point in space that we measure distances to
- different for every type of antenna
- ideally point in space, but depends on azimuth and elevation of signal
- models assume azimuthal independence, fit elevation

`http://www.geopp.de/gnpcv/`