GEOS F493 / F693
Geodetic Methods and Modeling

– Lecture 07: InSAR - Unwrapping the Phase –

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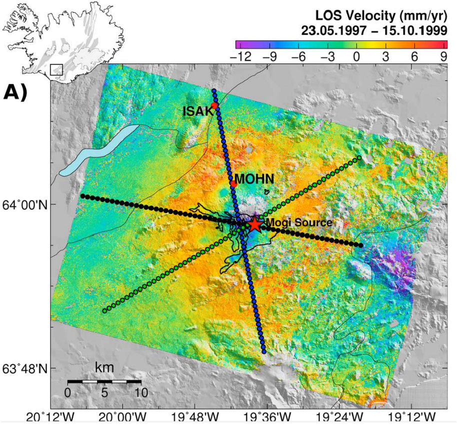
New Segment: “Guess the Process”



Grapenthin et al., 2010

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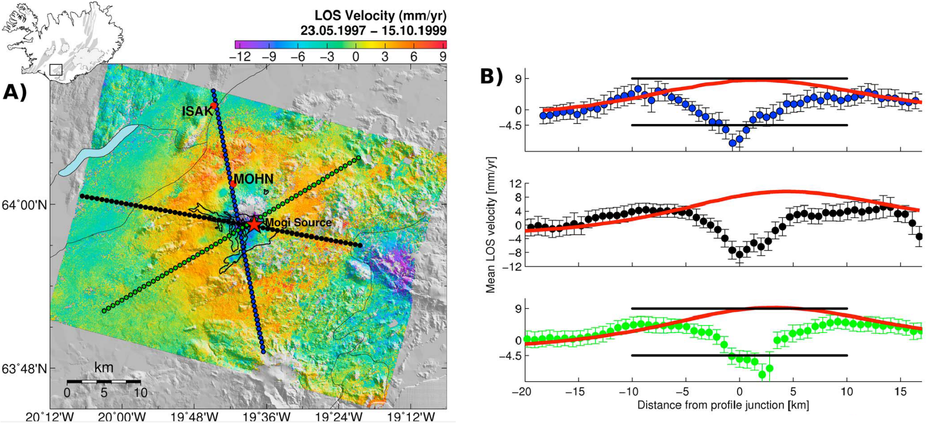


Grapenthin et al., 2010

InSAR - Processing Flow

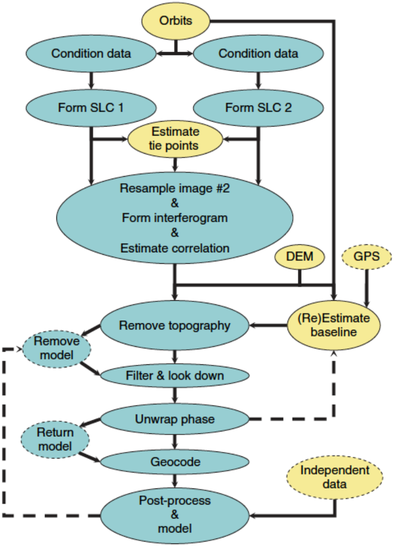
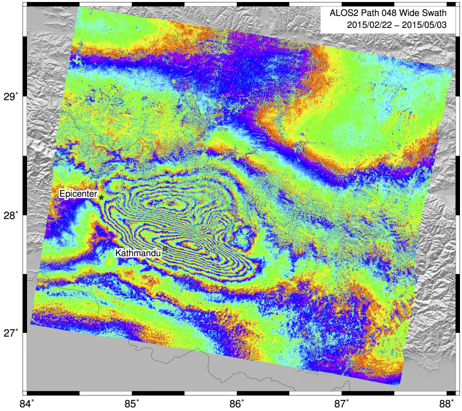


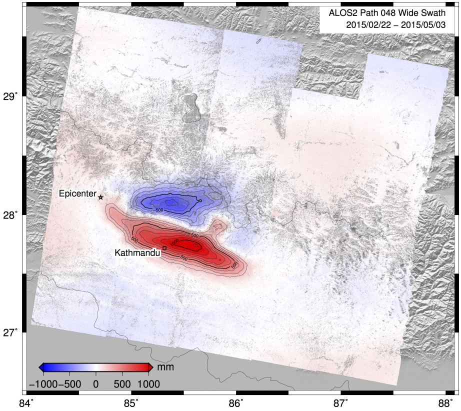
Figure 6 Representative differential InSAR processing flow diagram. Blue bubbles represent image output, yellow ellipses represent nonimage data. Flow is generally down the solid paths, with optional dashed paths indicating potential iteration steps. DEM, digital elevation model; SLC, single look complex image.

Simons and Rosen, 2007

Getting from here . . .



. . . to here



Lindsey et al., GRL, 2015

Materials for this lecture come mostly from:

- Goldstein, R., Zebker, H., and Werner, C. (1988). *Satellite radar interferometry- Two-dimensional phase unwrapping.* Radio science, 23(4), 713-720.
- Rosen, P., Hensley, S., Joughin, I. R., Li, F. K., Madsen, S. N., Rodriguez, E., and Goldstein, R. M. (2000). *Synthetic aperture radar interferometry.* Proceedings of the IEEE, 88(3), 333-382.
- Chen, C. W. and Zebker, H. A. (2001). *Two-dimensional phase unwrapping with use of statistical models for cost functions in nonlinear optimization.* JOSA A, 18(2), 338-351.
- Hooper, A. and Zebker, H. A. (2007). *Phase unwrapping in three dimensions with application to InSAR time series.* JOSA A, 24(9), 2737-2747.

InSAR - Phase Unwrapping

- remove modulo-2π ambiguity
- classes of algorithms:
  - integration with branch cuts
  - *L*-norm minimization (fit unwrapped solution to gradients of wrapped phase, minimize cost function)
  - mixed L-norms + probabilistic approach (snaphu)
  - 2D, 3D (where third dimension is time)

InSAR - Phase Unwrapping: Naive Approach

- assume neighboring phase values vary slowly: within one half-cycle (π rad)
- integrate phase differences from point to point
- add integer number of cycles that minimized phase differences
- 1D example (unit: cycles): 0.5, 0.6, 0.7, 0.8, 0.9, 0.0, 0.1, 0.2 . . .
- clearly need to add 1 cycle to last 3 values

What are possible unwrapping errors?

InSAR - Phase Unwrapping: Naive Approach

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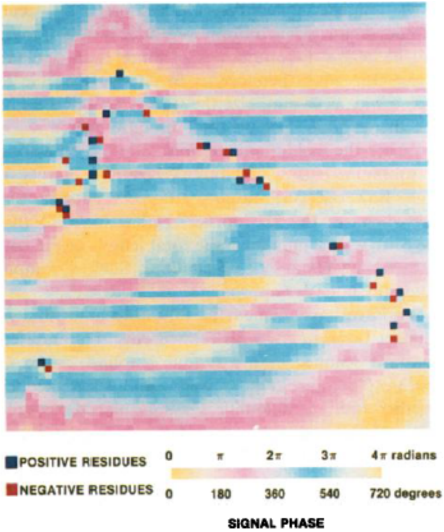
What are possible unwrapping errors?

- local errors: a few points are noise-corrupted
- global errors: local error propagates through sequence

Problem: Errors or phase variations $> \pi$ make integration path dependent!

InSAR - Phase Unwrapping: Naive Approach



Goldstein et al., JGR, 1988

InSAR - Phase Unwrapping: Branch Cut

- evaluate clock-wise sum of adjacent points:

$$\begin{array}{ccc} 0.0 & \rightarrow & 0.3 \\ \uparrow & & \downarrow \\ 0.8 & \leftarrow & 0.6 \end{array}$$

Goldstein et al., JGR, 1988

- zero $\pm$ 1 cycle if phase difference consistent with half-cycle assumption
- inconsistencies with half-cycle assumption indicated by non-zero results
- such “residues” are either positively or negatively “charged” (depending on sign of sum)

InSAR - Phase Unwrapping: Branch Cut

- integration paths that enclose **single residue** have **inconsistency** in unwrapped phase
- integration paths that enclose equal number of plus and minus residue have **no inconsistency**
- when residues identified: consistent unwrapping possible

InSAR - Phase Unwrapping: Branch Cut

- “branch cuts” between residues prevent integration path from crossing
- various (fully automated) strategies to choose cuts (e.g., minimize total discontinuity)

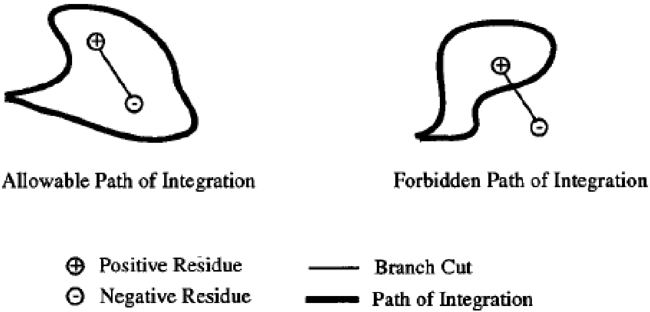
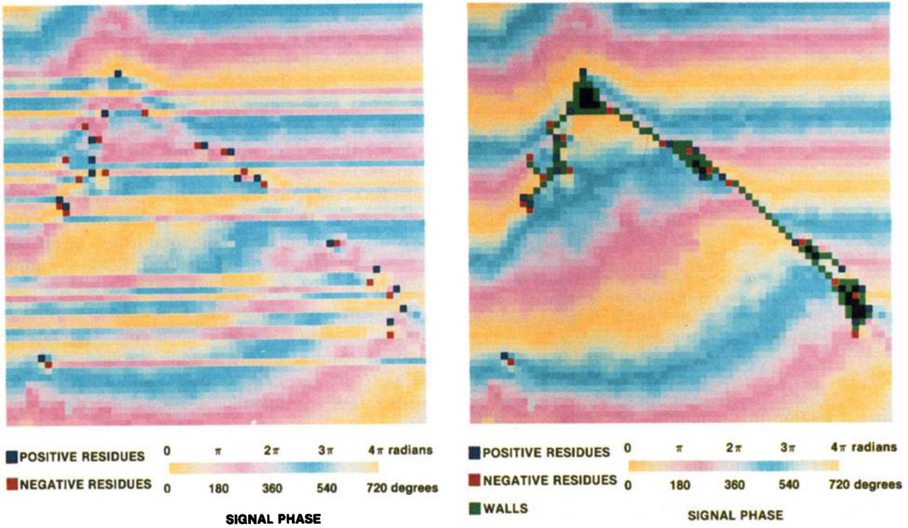


Fig. 18. An example of a branch cut and allowable and forbidden paths of integration.

Rosen et al., Proc. IEEE, 2000

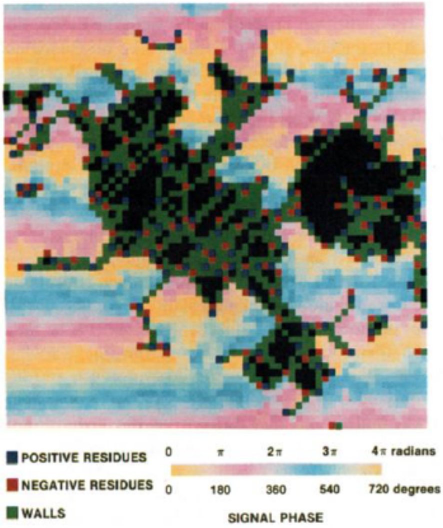
InSAR - Phase Unwrapping: Branch Cut



Goldstein et al., JGR, 1988

Cuts in place, not yet integrated

InSAR - Phase Unwrapping: Branch Cut



Goldstein et al., JGR, 1988

Dense area of residues: no reliable phase estimation possible, isolated from integration

Problem: How to select cuts?

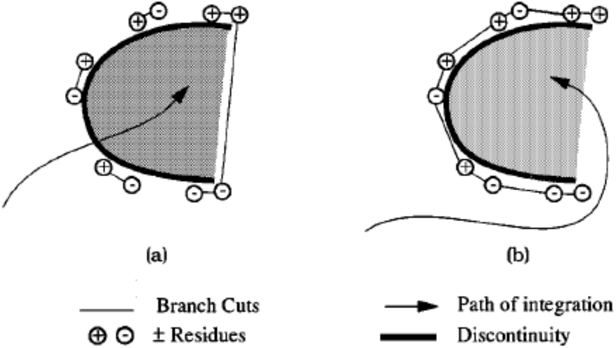


Fig. 19. Cut dependencies of unwrapped phase: (a) shortest path cuts and (b) better choice of cuts.

Rosen et al., Proc. IEEE, 2000

InSAR - Phase Unwrapping: L-norm minimization

Minimize (2D-range-azimuth coordinate system):

$$\sum_i \sum_j g_{ij}^{(r)}(\Delta\phi_{ij}^{(r)}, \Delta\psi_{ij}^{(r)}) + \sum_i \sum_j g_{ij}^{(a)}(\Delta\phi_{ij}^{(a)}, \Delta\psi_{ij}^{(a)})$$

InSAR - Phase Unwrapping: L-norm minimization

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- $\Delta\phi^{(r)}$, $\Delta\psi^{(r)}$: **range** component of wrapped, unwrapped (and rewrapped) phase gradients
- $\Delta\phi^{(a)}$, $\Delta\psi^{(a)}$: **azimuth** component of wrapped, unwrapped phase gradients
- e.g, $\Delta\phi_{ij}^{(r)} = \phi_{i,j} - \phi_{i-1,j}$, analog for azimuth, unwrapped components

InSAR - Phase Unwrapping: L-norm minimization

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Cost-function often restricted in form:

$$g_{ij}(\Delta\phi, \Delta\psi) = w_{ij}|\Delta\phi_{ij} - \Delta\psi_{ij}|^P$$

- all cost functions have same shape determined by constant P ($P = 2$: Least squares problem)
- indep. weights w determine each cost function's contribution

InSAR - Phase Unwrapping: L-norm minimization

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InSAR - Phase Unwrapping: L-norm & Probabilistic

- no physical reasons that optimal L^P solution must be correct
- *Chen & Zebker, JOSA, 2001* introduce objective from generalized, statistical cost functions
- allow any form for cost function g
- allow g shape to vary for different parts of interferogram
- choose cost function that maximizes conditional probably of solution based on wrapped phase, image intensity, coherence
- **application-specific** cost functions
- solution **approximation** based on non-linear network optimization