GEOS F493 / F693
Geodetic Methods and Modeling

– Lecture 11a: Modeling Slip on Faults and Regularization –

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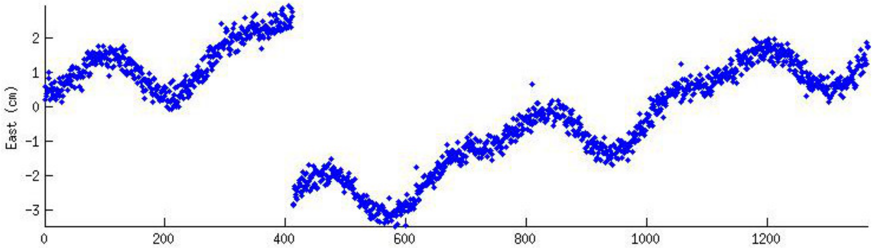
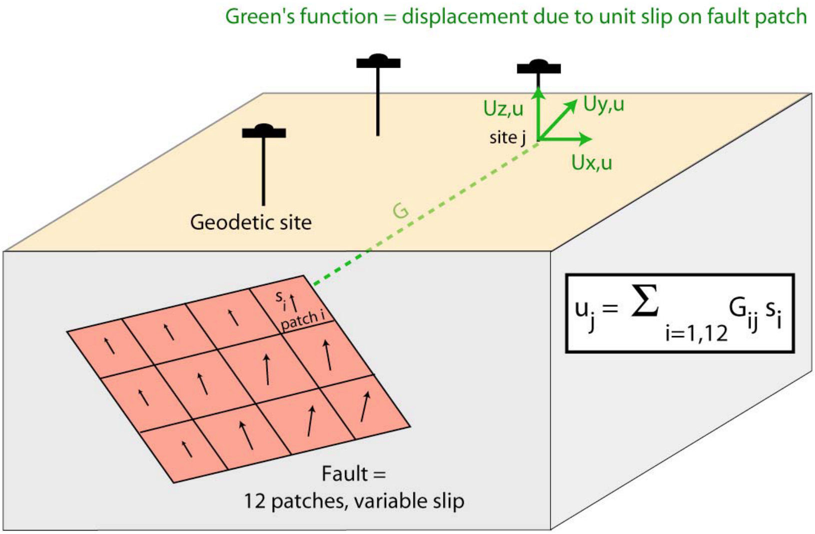
On modeling ...



...time (days) ...

Geodetic data → Slip on a Fault



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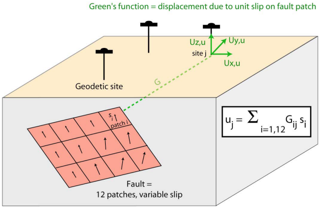
Green's Functions

- Is basically an impulse unit response
- Represents Earth structure ("effect of propagation from source to receiver")



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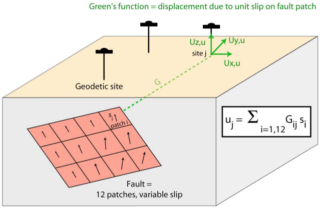
Green's Functions

- Is basically an impulse unit response
- Represents Earth structure ("effect of propagation from source to receiver")
- Think *"Given this Earth structure, how much displacement will I get here when the fault over there slips 1 unit (e.g., 1 m)"*
- Due to linearity (in slip amplitude) you can scale this with different amounts of slip, say 25 m or 33 cm which results in scaled displacement



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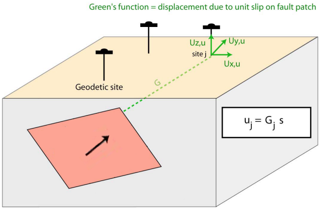
Green's Functions

- Simple earthquake: 1 fault surface with uniform strike dip, rake, slip
- Displacement at a location can be written as unit slip on that geometry times amount of slip



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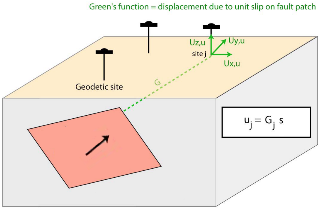
Green's Functions

- Simple earthquake: 1 fault surface with uniform strike dip, rake, slip
- Displacement at a location can be written as unit slip on that geometry times amount of slip

$$u = G * s$$

  - u is data vector
  - s is model vector
  - G is design matrix made of Green's functions
- G can be analytical expressions of derived from numerical models



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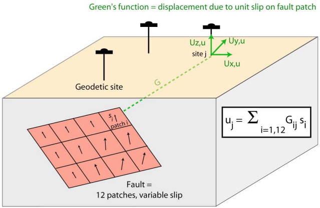
Green's Functions

- Complex earthquake: non-uniform strike dip, rake, slip
- complex fault geometry
- displacement at given site is sum of contributions of N fault patches

$$u_j = \sum_{i}^{N} G_{ij} * s_i$$



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Green's functions

Which primary directions of slip can we distinguish?

Green's functions

Which primary directions of slip can we distinguish?

- Strike-Slip (ss), Dip-Slip (ds), Opening (op)
- usually separated into their own Green's functions:

$$u_j = \sum_{i=1}^{N} \left[G_{ij}^{ss} s_i^{ss} + G_{ij}^{ds} s_i^{ds} + G_{ij}^{op} s_i^{op} \right]$$

Green's functions

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$$u_j = \sum_{i=1}^{N} \left[G_{ij}^{ss} s_i^{ss} + G_{ij}^{ds} s_i^{ds} + G_{ij}^{op} s_i^{op} \right]$$

- further separated into 3 displacement components:

$$u_j, x = \sum_{i=1}^{N} \left[G_{ij,x}^{ss} s_i^{ss} + G_{ij,x}^{ds} s_i^{ds} + G_{ij,x}^{op} s_i^{op} \right]$$

$$u_j, y = \sum_{i=1}^{N} \left[G_{ij,y}^{ss} s_i^{ss} + G_{ij,y}^{ds} s_i^{ds} + G_{ij,y}^{op} s_i^{op} \right]$$

$$u_j, z = \sum_{i=1}^{N} \left[G_{ij,z}^{ss} s_i^{ss} + G_{ij,z}^{ds} s_i^{ds} + G_{ij,z}^{op} s_i^{op} \right]$$

Green's functions

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- What kind of problem are we headed towards?

Green's Functions

- Analytical solution for elastic half-space exist
  - widely used formulation: *Okada, Y., Internal deformation due to shear and tensile faults in a half-space, Bull. Seismo. Soc. Amer., v. 82, 1018-1040, 1992.*
  - Original Fortran code is most reliable, implementations in other languages exist
- expressions for more complex earth structure exist
  - layered elastic
  - visco-elastic half space
  - elastic over visco-elastic

Solving for Slip

- Displacement at a point j on Earth's surface caused by slip on N fault patches can be written as:

$$u_j = \sum_{i=1}^{N} G_{ij} s_i$$

- This looks familiar

$$u = Gs$$

  - u is data vector
  - s is model vector
  - G is design matrix made of Green's functions

Solving for Slip

strike-slip | dip-slip

data vector (observed displacements)

$$
\begin{bmatrix} u_{e1} \\ u_{e2} \\ \cdots \\ u_{n1} \\ u_{n2} \\ \cdots \\ u_{u1} \\ u_{u2} \\ \cdots \end{bmatrix}
=
\left[\begin{array}{cccc:cccc}
g_1^{ss} & g_2^{ss} & \cdots & g_N^{ss} & g_1^{ds} & g_2^{ds} & \cdots & g_N^{ds} \\
g_1^{ss} & g_2^{ss} & \cdots & g_N^{ss} & g_2^{ds} & g_2^{ds} & \cdots & g_N^{ds} \\
 & & & & & & & \\
\end{array}\right]
\begin{bmatrix} s_{ss,1} \\ s_{ss,2} \\ \cdots \\ s_{ss,N} \\ s_{ds,1} \\ s_{ds,2} \\ \cdots \\ s_{ds,N} \end{bmatrix}
$$

Vector of estimates (slip on each patch)

patch 1 | patch N

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Solving for Slip

$$
\begin{array}{c}
\text{data vector} \\ \text{(observed} \\ \text{displacements)}
\end{array}
\rightarrow
\begin{bmatrix} u_{e1} \\ u_{e2} \\ \cdots \\ u_{n1} \\ u_{n2} \\ \cdots \\ u_{u1} \\ u_{u2} \\ \cdots \end{bmatrix}
=
\begin{array}{c}
\overleftrightarrow{\text{strike-slip}} \qquad\qquad \overleftrightarrow{\text{dip-slip}} \\
\left[\begin{array}{cccc:cccc}
g_1^{ss} & g_2^{ss} & \cdots & g_N^{ss} & g_1^{ds} & g_2^{ds} & \cdots & g_N^{ds} \\
g_1^{ss} & g_2^{ss} & \cdots & g_N^{ss} & g_2^{ds} & g_2^{ds} & \cdots & g_N^{ds} \\
 & & & & & & & \\
 & & & & & & &
\end{array}\right]
\end{array}
\begin{bmatrix} s_{ss,1} \\ s_{ss,2} \\ \cdots \\ s_{ss,N} \\ s_{ds,1} \\ s_{ds,2} \\ \cdots \\ s_{ds,N} \end{bmatrix}
\leftarrow
\begin{array}{l}
\text{Vector of} \\ \text{estimates} \\ \text{(slip on each} \\ \text{patch)}
\end{array}
$$

patch 1 patch N

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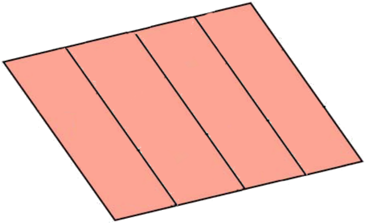
For prior 1D problems *G* was a matrix
How to deal with 2D problem of slip on fault?

Solving for Slip

This should be straight-forward to turn into G



Solving for Slip

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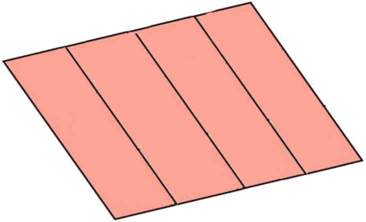
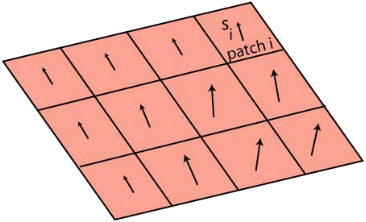


How about this?



Solving for Slip

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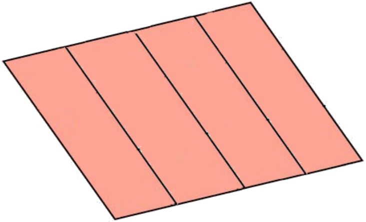

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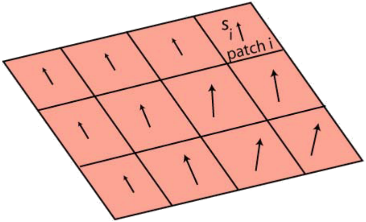

Linearize!

Solving for Slip

strike-slip | dip-slip

data vector (observed displacements)

$$
\begin{bmatrix} u_{e1} \\ u_{e2} \\ \cdots \\ u_{n1} \\ u_{n2} \\ \cdots \\ u_{u1} \\ u_{u2} \\ \cdots \end{bmatrix}
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 & & & & & & & \\
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\begin{bmatrix} s_{ss,1} \\ s_{ss,2} \\ \cdots \\ s_{ss,N} \\ s_{ds,1} \\ s_{ds,2} \\ \cdots \\ s_{ds,N} \end{bmatrix}
$$

Vector of estimates (slip on each patch)

patch 1 | patch N

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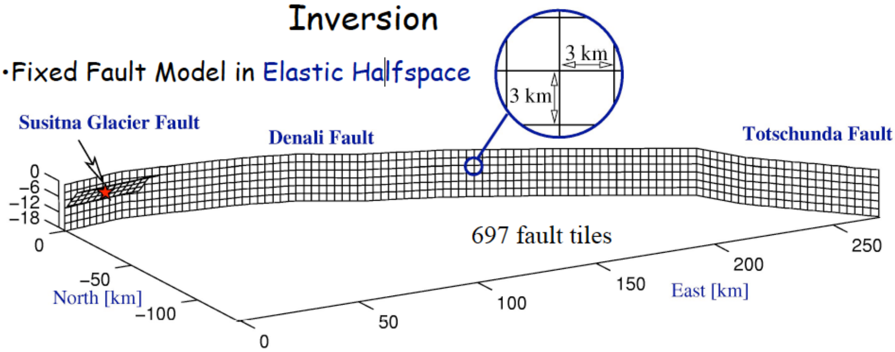
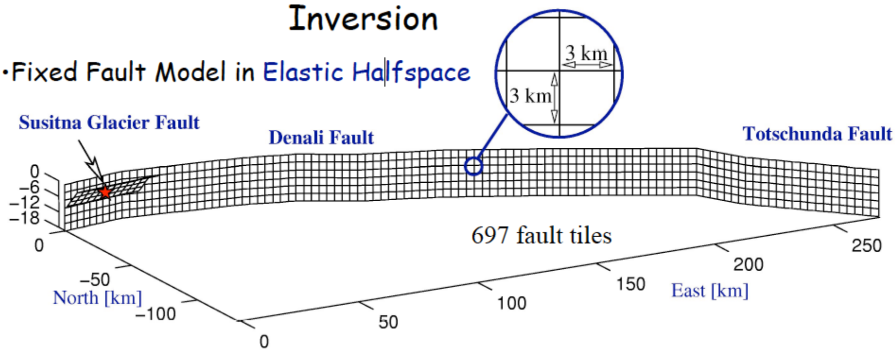
Solving for Slip



Sigrun Hreinsdottir

With 224 GPS sites and 697 fault tiles solving for dip-slip and strike-slip, what problem are we running into?

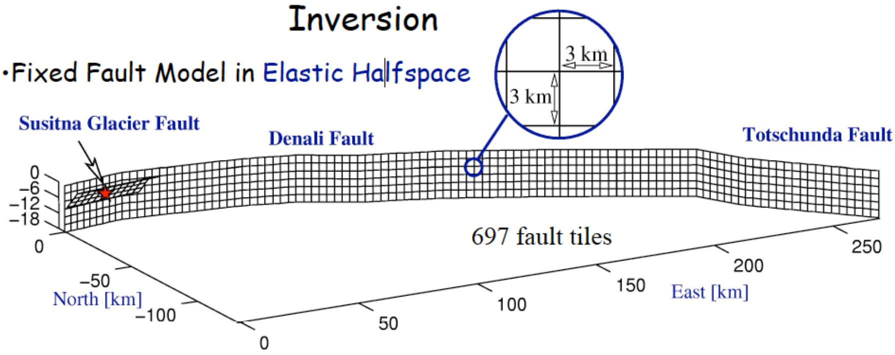
Solving for Slip



Sigrun Hreinsdottir

With 224 GPS sites and 697 fault tiles solving for dip-slip and strike-slip, what problem are we running into?
Underdetermined system.

Solving for Slip



Sigrun Hreinsdottir

- observations at 225 GPS sites: 675 data (if vertical helps)
- 697 fault tiles, ss, ds: 1394 unknowns
- no enough data to constrain number of unknowns
- also often an issue: unphysical oscillatory slip

Regularization / Smoothing

- Idea: Minimize the rate of change of slip with position
- "rate of change of slip" is curvature
- Laplacian:

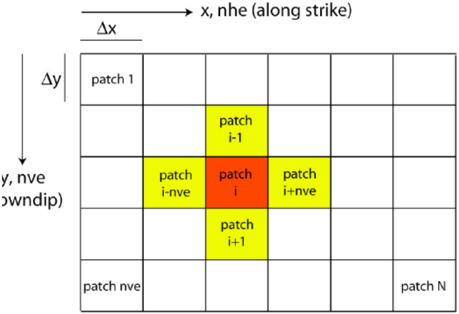
$$\nabla^2 = \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2}$$

- Practice: Minimize sum of partial second differentials of slip for each fault patch
- Can be solved using finite-difference method for a function P

$$\frac{\delta^2 P(x)}{\delta x^2} \approx \frac{P(x - \Delta x) - 2P(x) + P(x - \Delta x)}{\Delta x^2}$$

Regularization / Smoothing

- Our function $P(x)$ is slip s which varies along-strike (x) and down-dip (y)
- For patch i finite difference approximation of Laplacian is (nve = number of vertical elements, nhe = horiztonal):



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$$l_i = \frac{s_{i-nve} - 2s_i + s_{i+nve}}{\Delta x^2} + \frac{s_{i-1} - 2s_i + s_{i+1}}{\Delta x^2}$$

Regularization / Smoothing

- In practice, equation:

$$l_i = \frac{s_{i-nve} - 2s_i + s_{i+nve}}{\Delta x^2} + \frac{s_{i-1} - 2s_i + s_{i+1}}{\Delta y^2}$$

is written in matrix form, for the along-strike and down-sip components:

$$\begin{bmatrix} \dots \\ 0 \\ \dots \end{bmatrix} = \underbrace{\begin{bmatrix} \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ (0) & \frac{1}{\Delta x^2} & (0) & -\frac{2}{\Delta x^2} & (0) & -\frac{1}{\Delta x^2} & (0) \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}}_{L_x} \begin{bmatrix} \dots \\ s_{i-nve} \\ \dots \\ s_i \\ \dots \\ s_{i+nve} \\ \dots \end{bmatrix} \qquad \begin{bmatrix} \dots \\ 0 \\ \dots \end{bmatrix} = \underbrace{\begin{bmatrix} \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ (0) & \frac{1}{\Delta y^2} & (0) & -\frac{2}{\Delta y^2} & (0) & -\frac{1}{\Delta y^2} & (0) \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}}_{L_y} \begin{bmatrix} \dots \\ s_{i-1} \\ \dots \\ s_i \\ \dots \\ s_{i+1} \\ \dots \end{bmatrix}$$

- The 2 Laplacian matrices are then added:

$$L = L_x + L_y$$

Regularization / Smoothing

- The original problem was:

$$[u] = \begin{bmatrix} G_{ss} & G_{ds} \end{bmatrix} \begin{bmatrix} s_{ss} \\ s_{ds} \end{bmatrix}$$

- now it becomes:

$$\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} G_{ss} & G_{ds} \\ L & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} s_{ss} \\ s_{ds} \end{bmatrix}$$

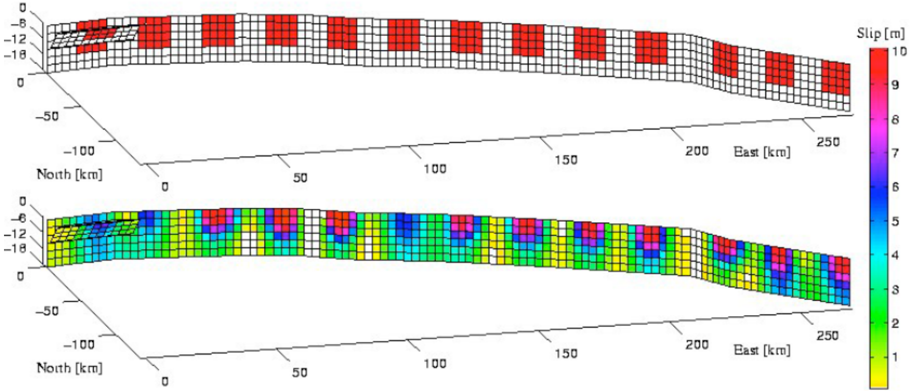
- amount of smoothing can be tuned using scalar smoothing factor κ:

$$\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} G_{ss} & G_{ds} \\ \kappa L & 0 \\ 0 & \kappa L \end{bmatrix} \begin{bmatrix} s_{ss} \\ s_{ds} \end{bmatrix}$$

- $\kappa = 0$: no smoothing, $\kappa = 1$ maximum smoothing

Regularization / Smoothing

What can you recover? Checker board / Resolution test:



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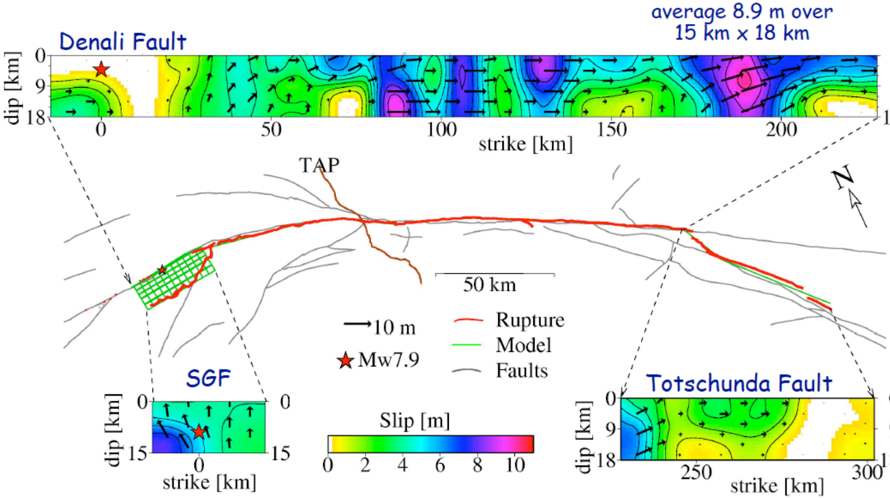
Distributed Slip Inversion

This is how you get this:



M_0=6.81x10^{20}Nm
M_w7.89

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