



# GEOS F493 / F693

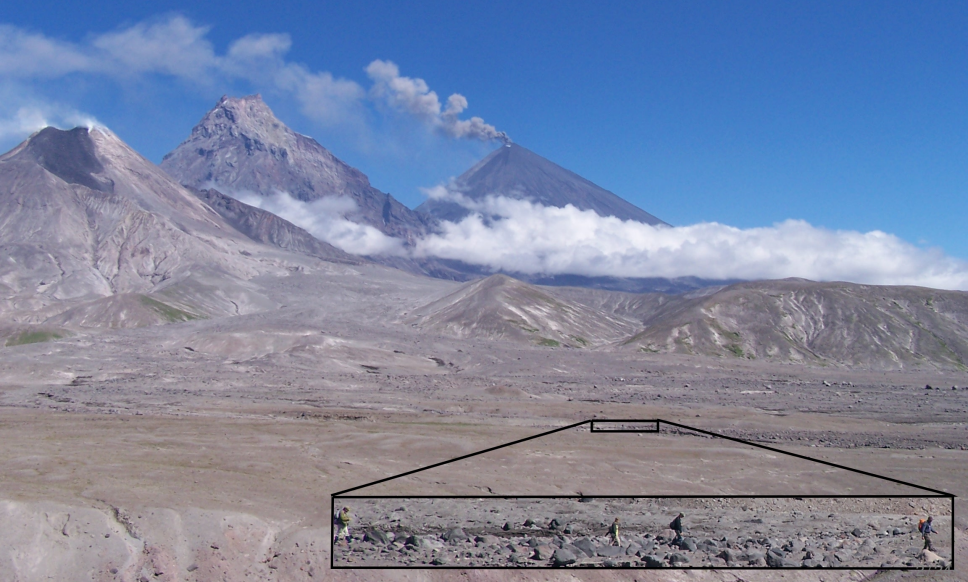
## Geodetic Methods and Modeling

### – Lecture 11b: Modeling - Volcano Deformation –

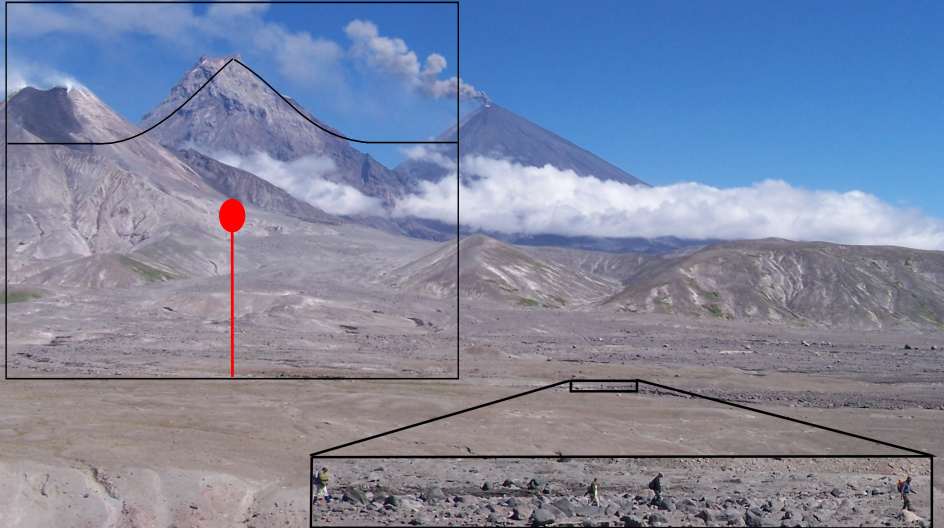
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Elvey 413B  
(907) 474-7286

November 11, 2019

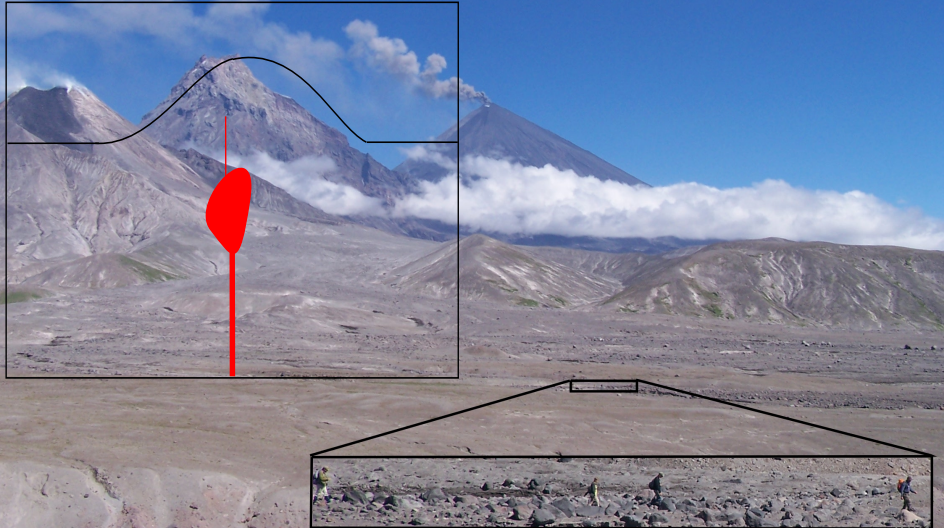
# 1. Plumbing: How does GPS help?



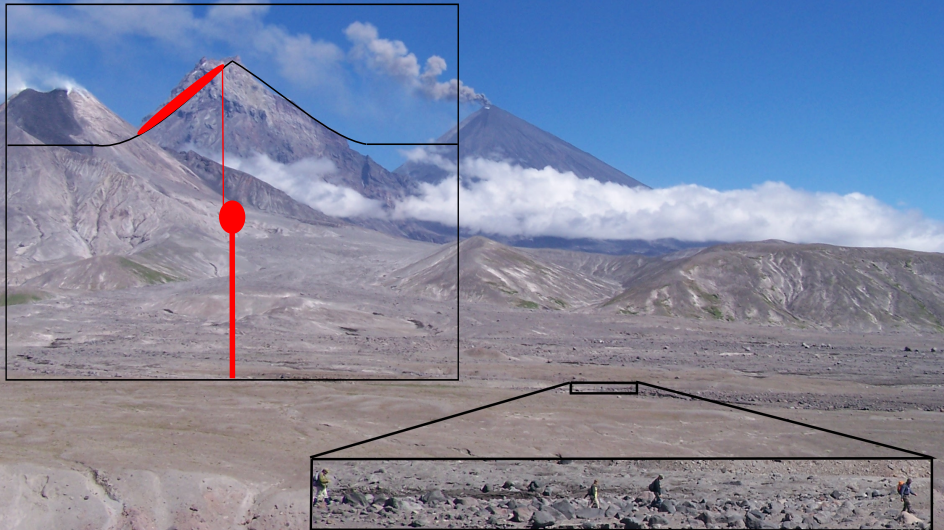
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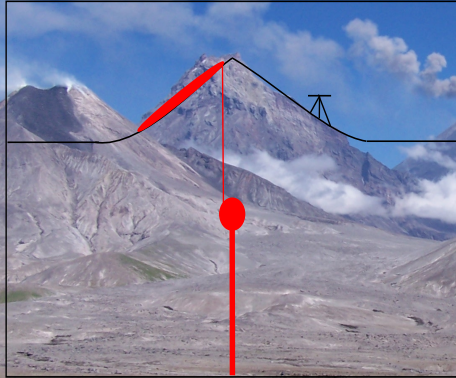
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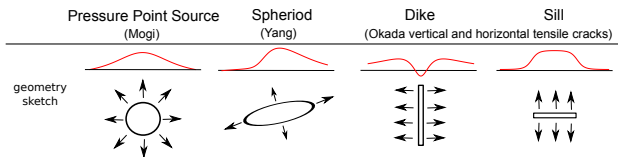
# 1. Plumbing: How does GPS help?



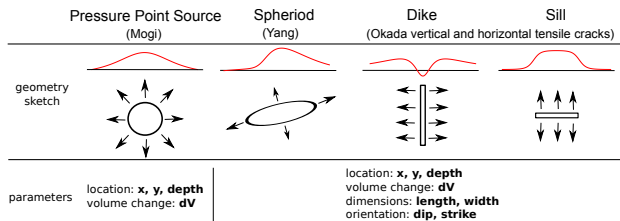
- Geodetic tools measure deformation: GPS, InSAR, ...
- Analytical models link deformation to volcano source characteristics



# Source Models

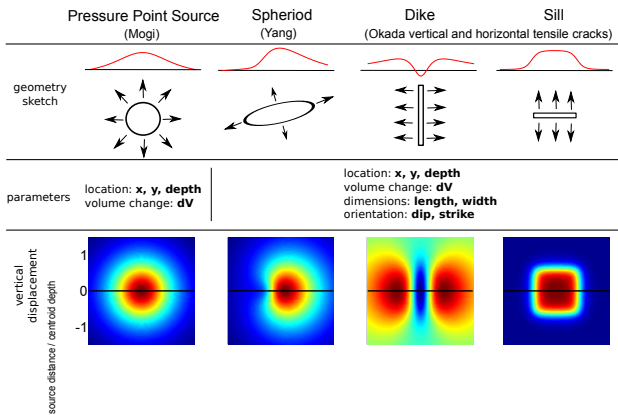


# Source Models

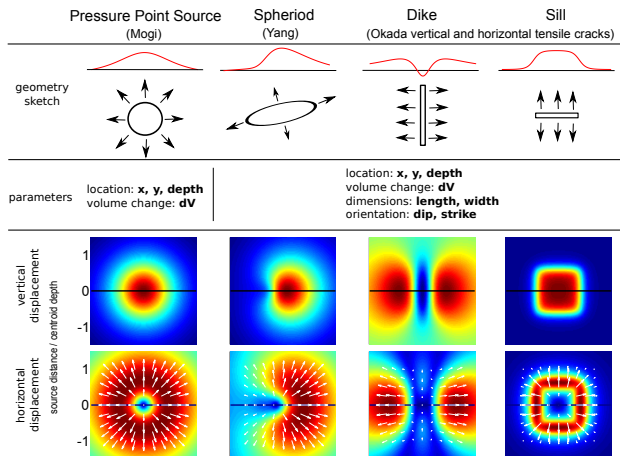




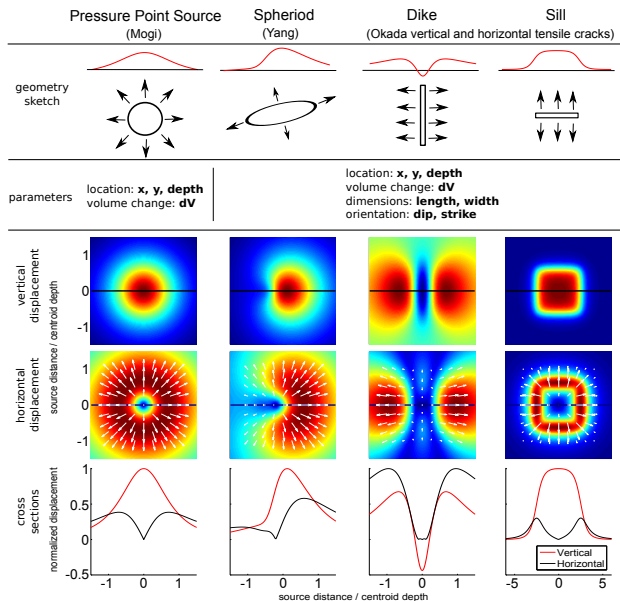
# Source Models



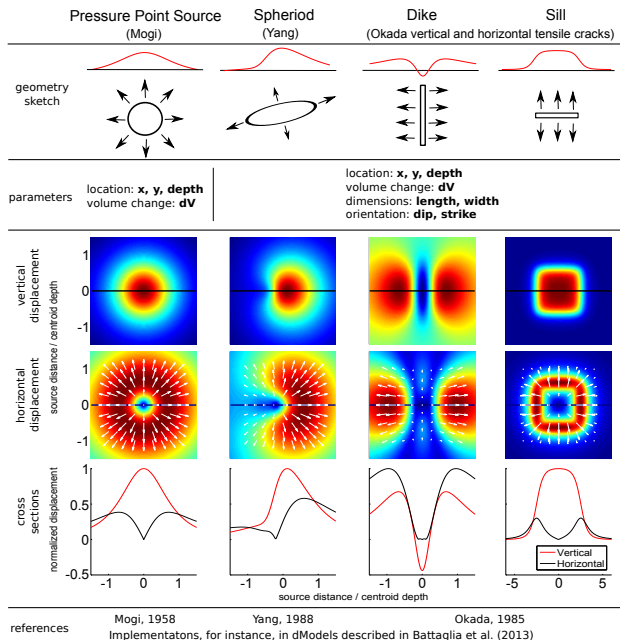
# Source Models



# Source Models

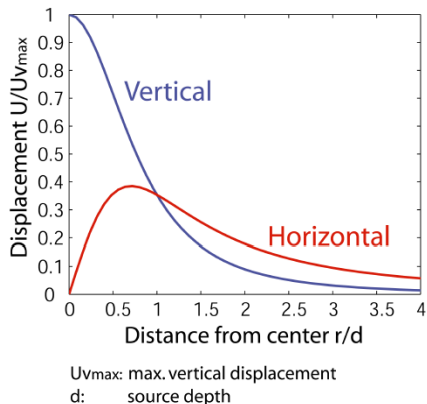
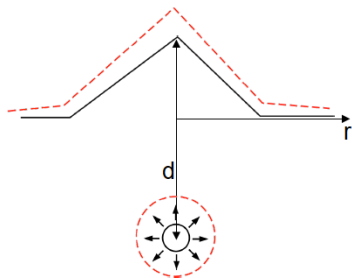


# Source Models



# Source Models: Mogi (1958)

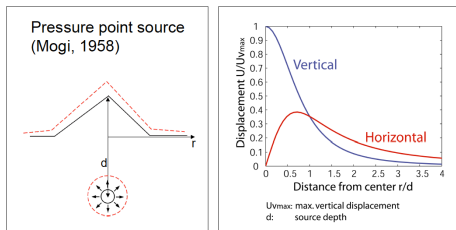
Pressure point source  
(Mogi, 1958)



Jeff Freymueller

Model parameters: lat, lon, depth, source strength

# Source Models: Mogi (1958)



Jeff Freymueller

- $r$  - radial distance from source
- $d$  - source depth
- $\nu$  - Poisson's ratio (0.25)
- $C = \frac{(1-\nu)\Delta V}{\pi}$  - source strength

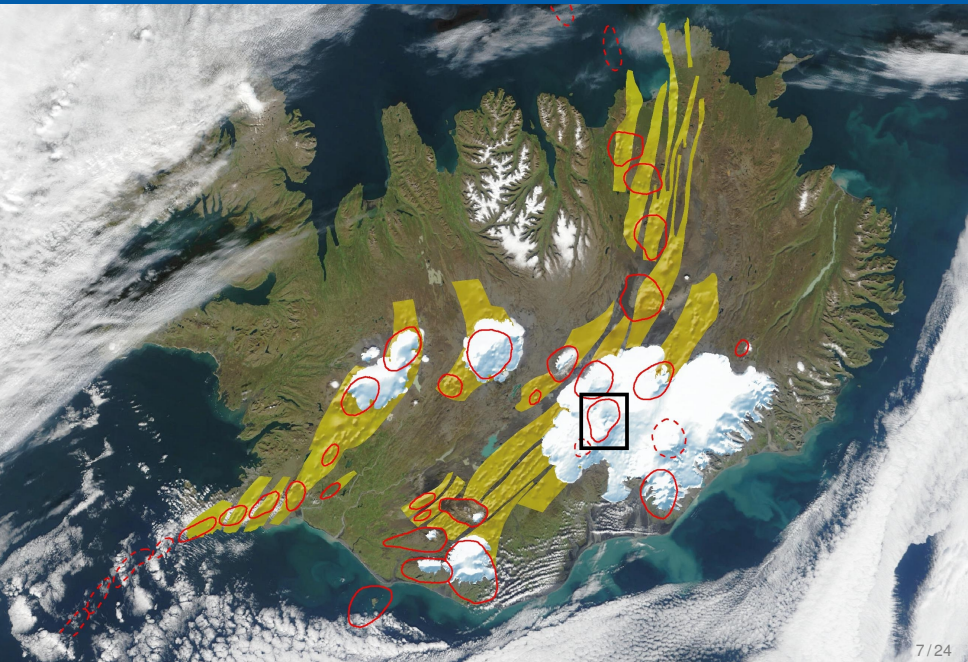
$$u_z = \frac{(1-\nu)\Delta V}{\pi} \frac{d}{(r^2 + d^2)^{3/2}}$$
$$u_r = \frac{(1-\nu)\Delta V}{\pi} \frac{r}{(r^2 + d^2)^{3/2}}$$

- $\Delta V = \frac{\rho a^3}{\mu}$  - source volume change (see later!)
- $\rho$  - pressurization
- $a$  - source radius
- $\mu$  - shear modulus

## Source Models: Mogi (1958)

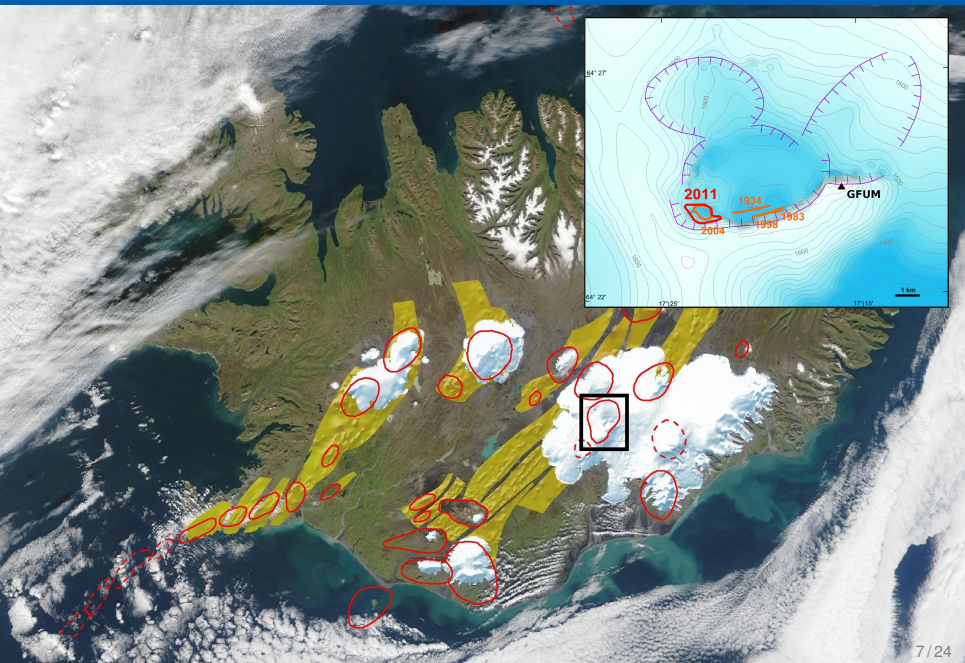
- $\Delta V$  is volume change of the chamber  $\neq$  magma volume change
- equivalent to scaled pressure change in cavity
- doesn't consider magma compressibility (more compressible the more gases are exsolved)
- volume is function of pressure and mass
- point source approximation means  $a \ll d$ , in practice good approx. for  $a < 0.5d$

# 2011 Grímsvötn

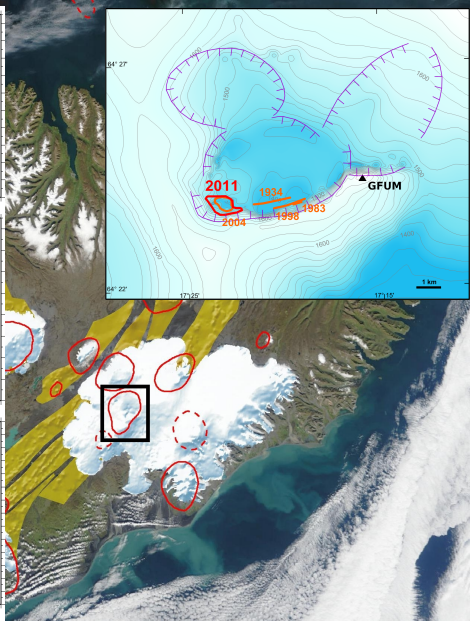
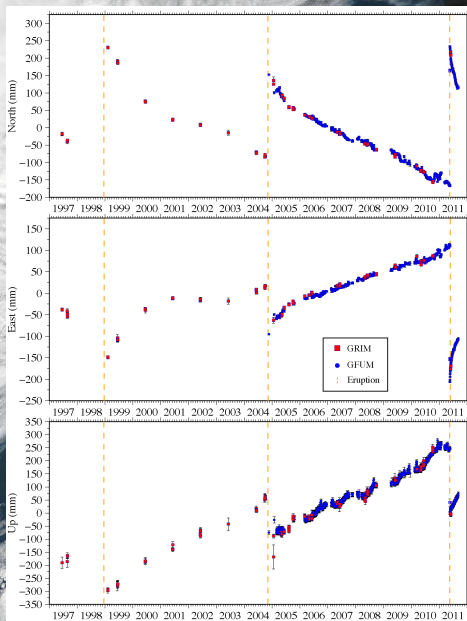




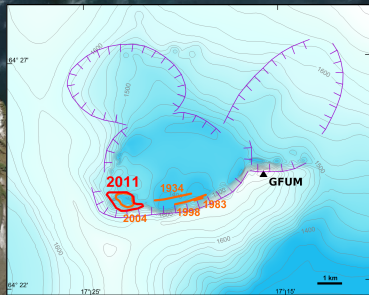
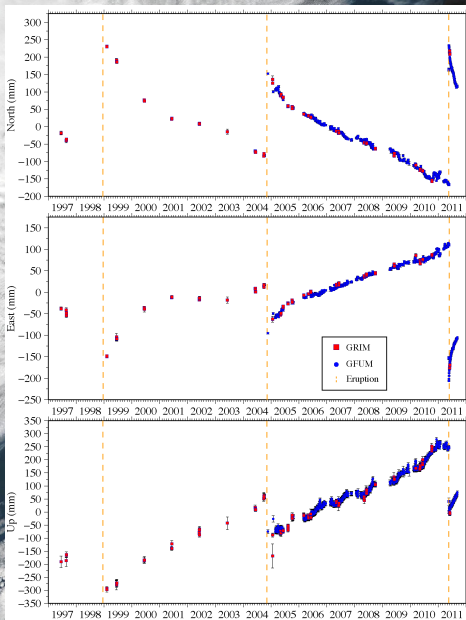
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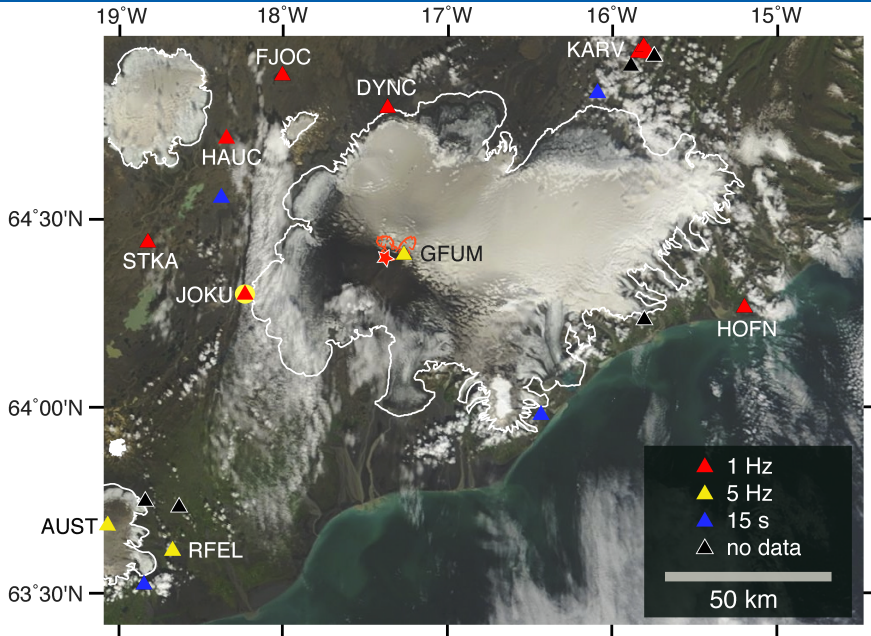


# 2011 Grímsvötn



- explosive eruption  
21-28 May 2011
- plumes > 20 km
- continuous inflation,  
gradual increase in  
seismicity

# 2011 Grímsvötn: Geodetic Network



# 2011 Grímsvötn: GPS Processing

## 1 Hz:

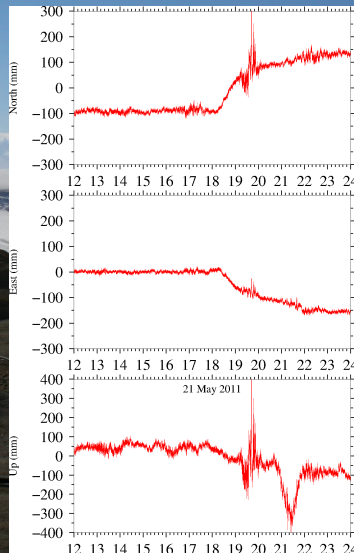
- Kinematic trajectories from track (GAMIT/GLOBK, MIT)
- Search for closest, unbiased sites
- 7 base stations about 50-100 km away
- Noise elimination: stacking of base lines
- Multi-path elimination with sidereal filtering: subtract pre-eruptive solutions shifted by 246 s
- more smoothing (15 s, 5 min windows)



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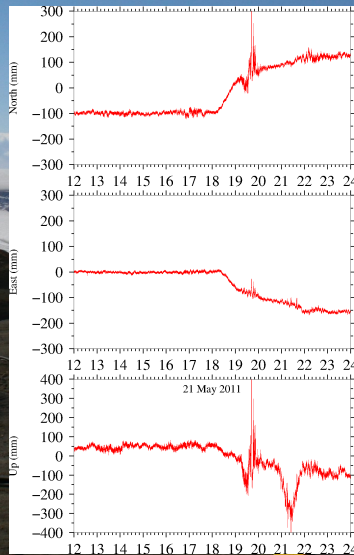


GFUM wrt JOKU

# 2011 Grímsvötn: GPS Processing

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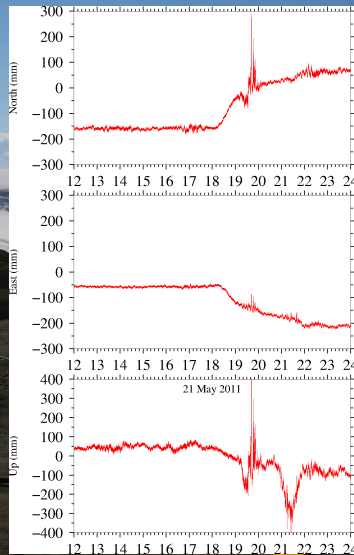


3 Station Stack

# 2011 Grímsvötn: GPS Processing

## 1 Hz:

- Kinematic trajectories from track (GAMIT/GLOBK, MIT)
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- more smoothing (15 s, 5 min windows)



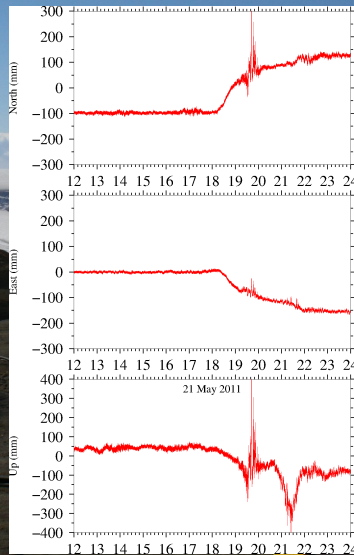
7 Station Stack



# 2011 Grímsvötn: GPS Processing

## 1 Hz:

- Kinematic trajectories from track (GAMIT/GLOBK, MIT)
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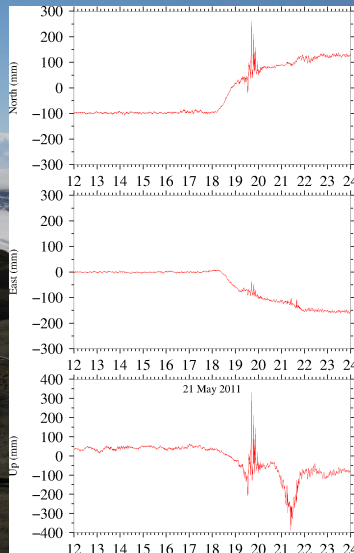


Sidereal Filtering

# 2011 Grímsvötn: GPS Processing

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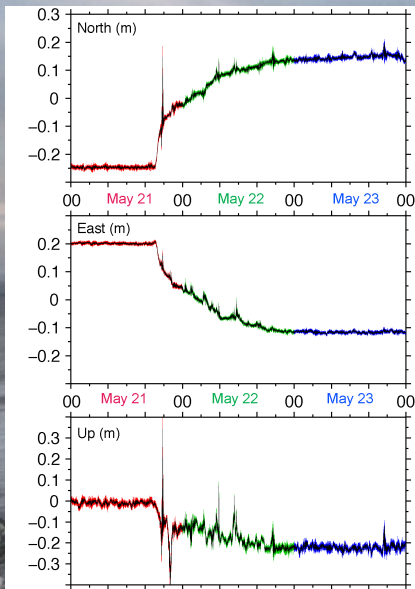


15 s Filter

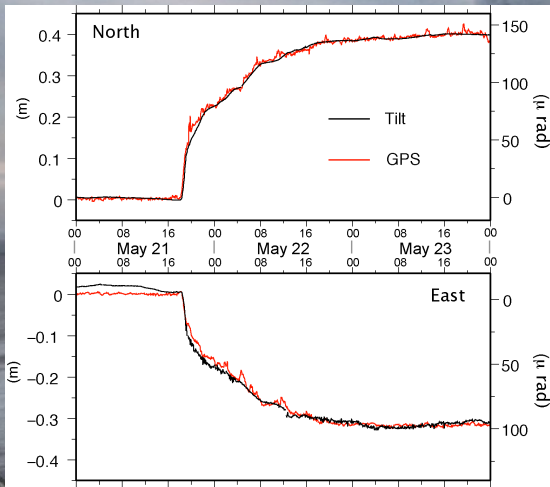
# 2011 Grímsvötn: Results



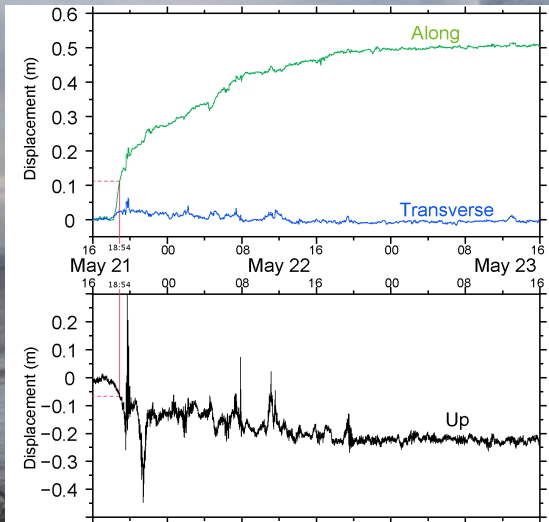
# 2011 Grímsvötn: Results



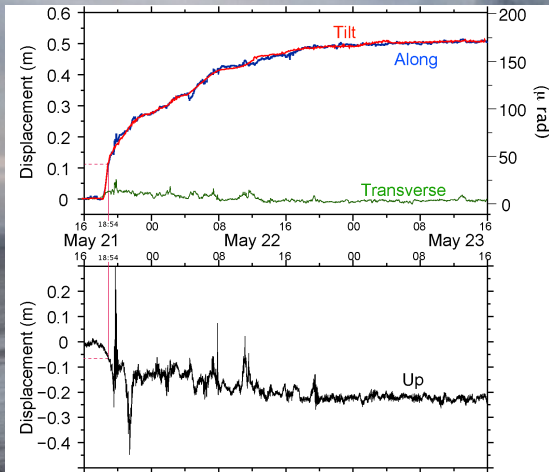
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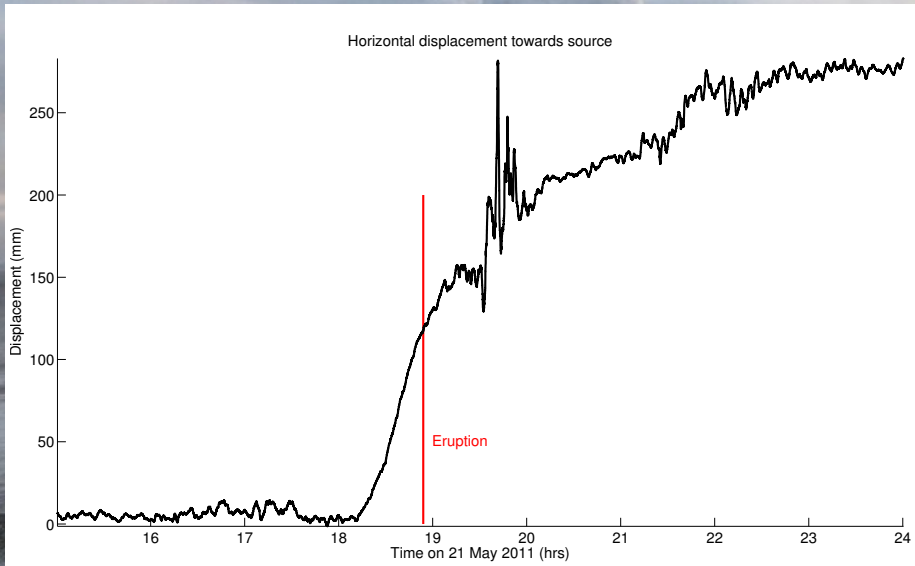
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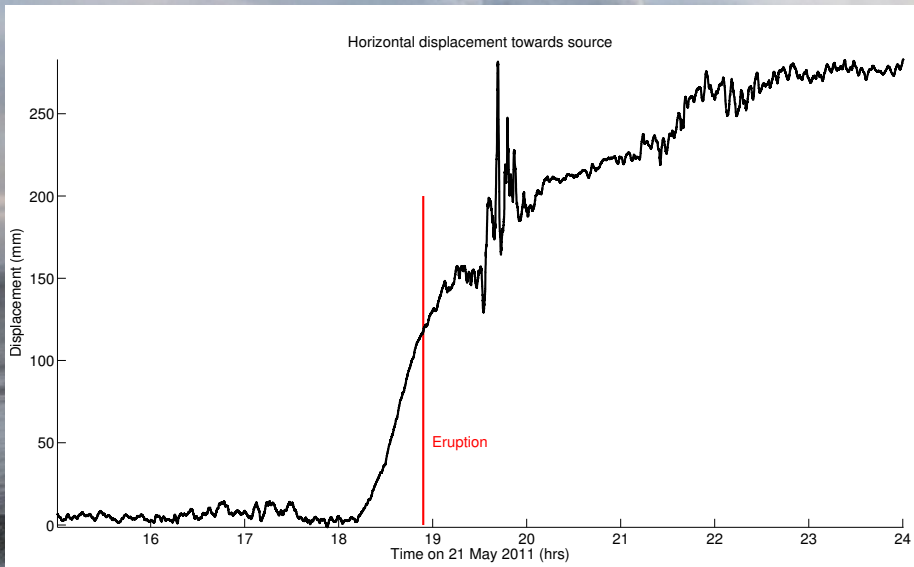


# 2011 Grímsvötn: Results

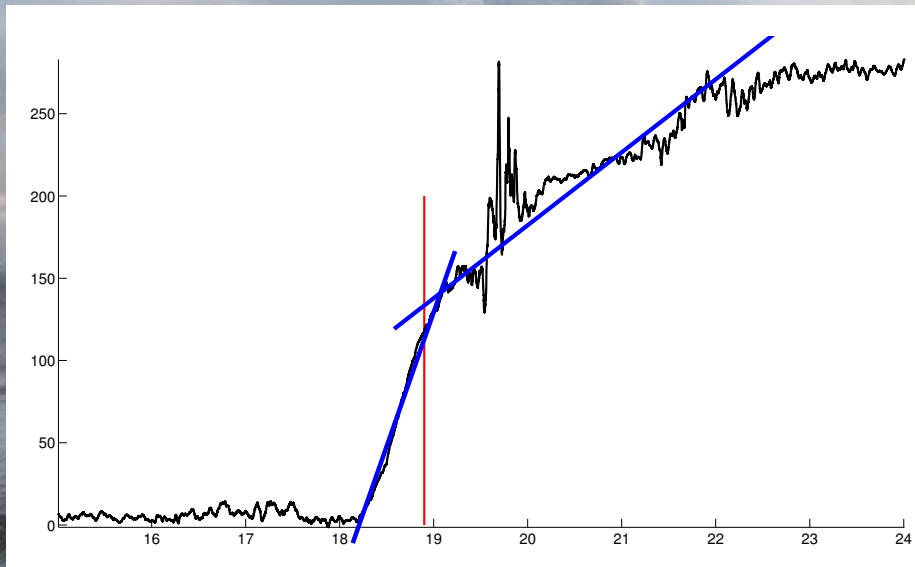




# 2011 Grímsvötn: Results



# 2011 Grímsvötn: Results



# 2011 Grímsvötn: Source Model

Measurements (Model given: Mogi 1958, Sigmundsson 2006):

$$\text{radial displacement} = u_r = C \frac{r}{(d^2 + r^2)^{3/2}} = \mathbf{51 \text{ cm N}38.5^\circ\text{W}}$$

$$\text{vertical displacement} = u_z = C \frac{d}{(d^2 + r^2)^{3/2}} = \mathbf{-25 \text{ cm}}$$

$$\text{tilt} = \delta = C \frac{-3dr}{(d^2 + r^2)^{5/2}} = \mathbf{171 \mu\text{rad}}$$

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$$\text{tilt} = \delta = C \frac{-3dr}{(d^2 + r^2)^{5/2}} = \mathbf{171 \mu\text{rad}}$$

We derive ( $1\sigma$  uncertainties):

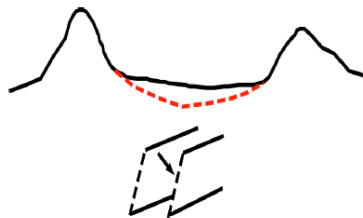
$$\text{distance} = r = \frac{u_r}{u_d} d = \mathbf{3.6 \pm 0.3 \text{ km}}$$

$$\text{depth} = d = -\frac{3u_z}{\delta} \frac{u_r/u_z}{1 + (u_r/u_z)^2} = \mathbf{1.8 \pm 0.2 \text{ km}}$$

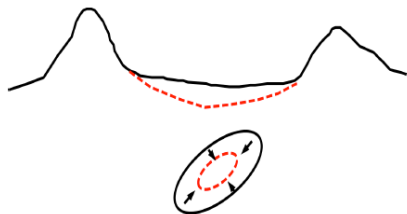
$$\text{strength} = C = 9 \frac{u_z^3 (u_r/u_z)^2}{\delta^2 \sqrt{1 + (u_r/u_z)^2}} = \mathbf{9 \pm 1 \times 10^6 \text{ m}^3}$$

# Source Models: Okada (1985), Yang (1988)

Rectangular dislocation source (Okada, 1985)



Ellipsoidal pressure source (Yang, 1988)



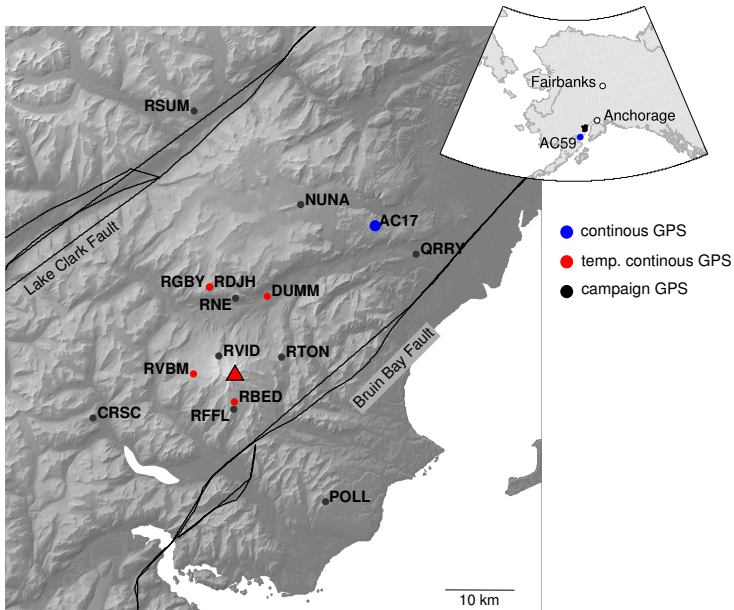
*Jeff Freymueller*

Model parameters: lat, lon, depth, length, width, dip, strike, source strength

# Mt Redoubt, Alaska, 2009



# 1. Weeks to Months: Mt. Redoubt Source Models

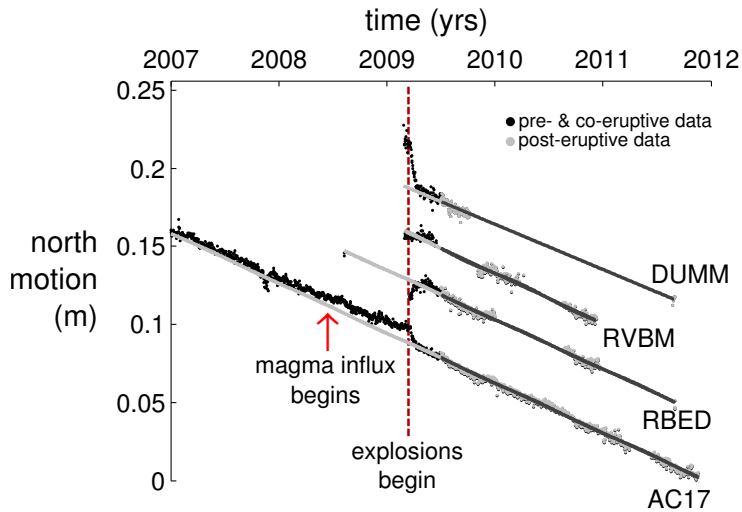


## Source Estimation:

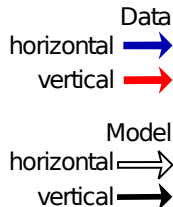
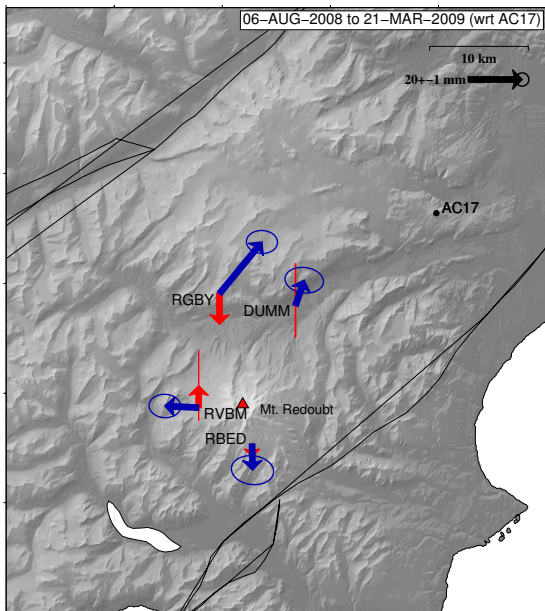
- Pressure Point Source (Mogi, 1958)
- degenerate prolate spheroid / conduit (Bonaccorso and Davis, 1999)
- general (prolate) spheroid (Yang 1986, Newman et al. 2006, Battaglia et al. 2012)
- Grid search over spatial domain (models non-linear in space)
- Least squares inversion for volume change



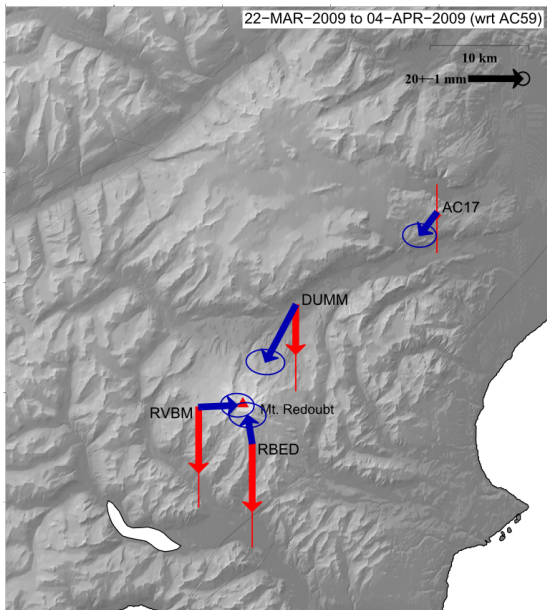
# GPS Time Series relative to North America



# Pre-eruptive Phase – Inflation

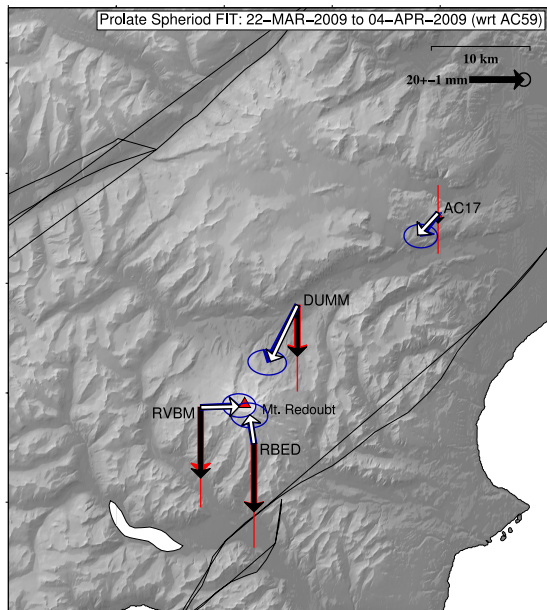


# Explosive Phase – Deflation



# Explosive Phase – Deflation

Prolate Spheroid FIT: 22-MAR-2009 to 04-APR-2009 (wrt AC59)



## General Spheroid:

$$r = 0.5 \text{ km E of dome}$$



$$d = 9.17 \begin{matrix} 6.92 \\ 15.17 \end{matrix} \text{ km}$$



$$a = 4.50 \begin{matrix} 1.25 \\ >10.00 \end{matrix} \text{ km}$$

$$b = 0.475 \begin{matrix} 0.3 \\ >4.00 \end{matrix} \text{ km}$$

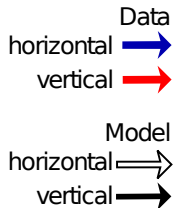
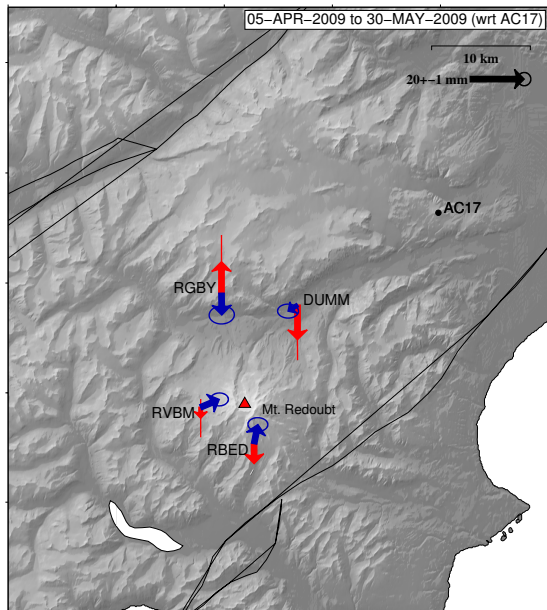
$$\Delta V = -(0.05 \begin{matrix} 0.028 \\ >0.1 \end{matrix}) \text{ km}^3$$

F-Test: Spheroid preferred.

Data  
horizontal   
vertical 

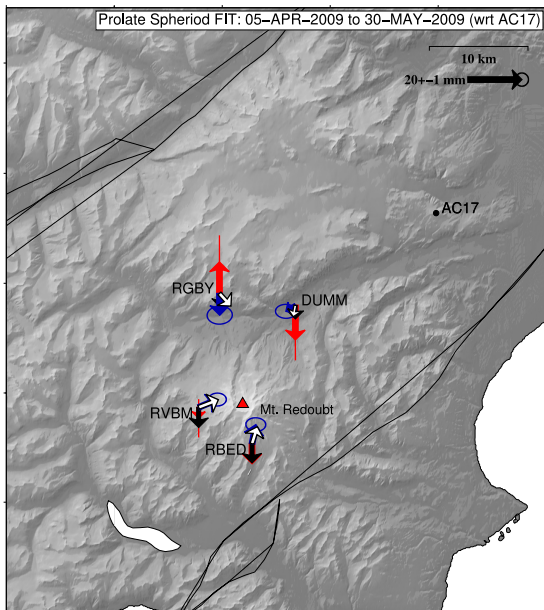
Model  
horizontal   
vertical 

# Effusive Phase – Deflation



# Effusive Phase – Deflation

Prolate Spheroid FIT: 05-APR-2009 to 30-MAY-2009 (wrt AC17)

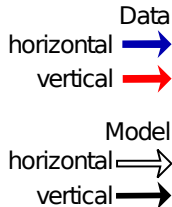


## General Spheroid:

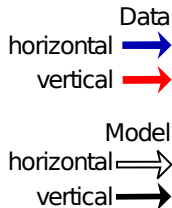
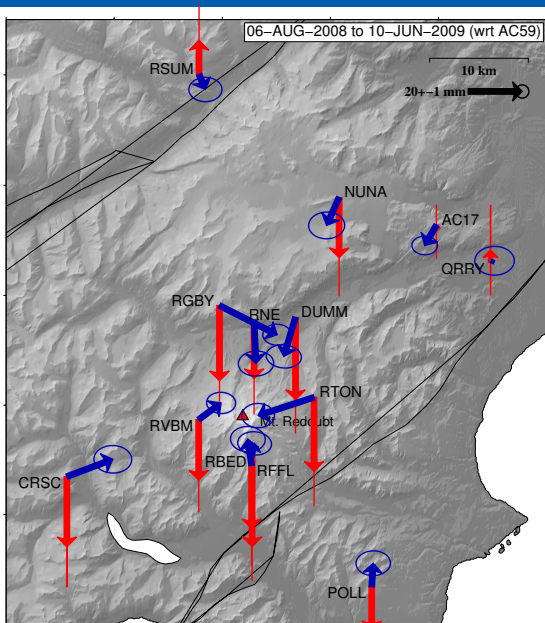
$$\Delta V = -(0.017^{0.011}_{0.023}) \text{ km}^3$$

Mogi fits better

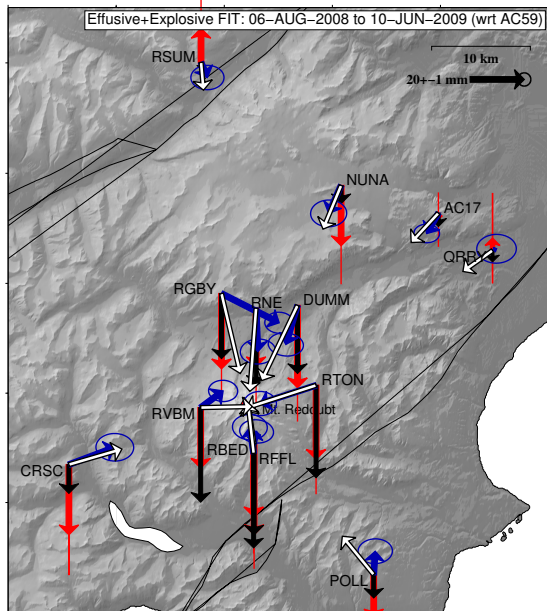
F-Test rejects Mogi



# Full Eruption – Net Deflation



# Full Eruption – Net Deflation



## Explosive: Prolate Spheroid

$$r = 0.5 \text{ km E of dome}$$

$$d = 9.17^{6.92}_{15.17} \text{ km}$$

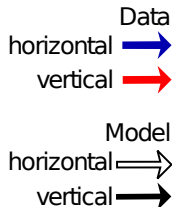
$$a = 4.50^{1.25}_{>10.00} \text{ km}$$

$$b = 0.475^{0.3}_{>4.00} \text{ km}$$

$$\Delta V = -(0.05^{0.028}_{>0.1}) \text{ km}^3$$

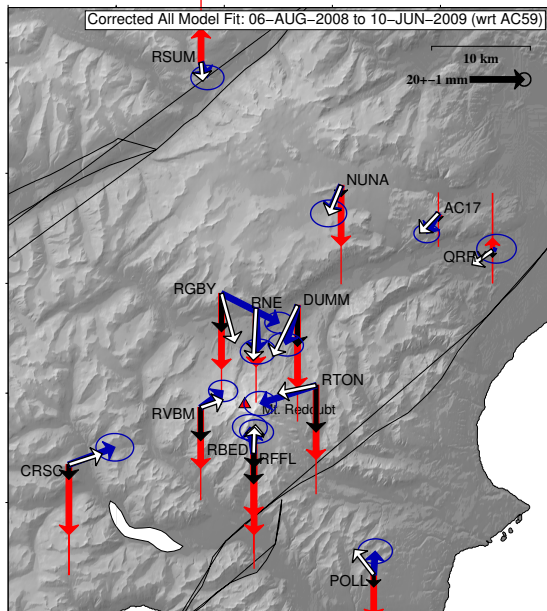
## Effusive: same.

$$\Delta V = -(0.017^{0.011}_{0.023}) \text{ km}^3$$





# Final Model



## Explosive: Prolate Spheroid

$$r = 0.5 \text{ km E of dome}$$

$$d = 9.17^{6.92}_{15.17} \text{ km}$$

$$a = 4.50^{1.25}_{>10.00} \text{ km}$$

$$b = 0.475^{0.3}_{>4.00} \text{ km}$$

$$\Delta V = -(0.05^{0.028}_{>0.1}) \text{ km}^3$$

## Effusive: same.

$$\Delta V = -(0.017^{0.011}_{0.023}) \text{ km}^3$$

## Pre-eruptive: Mogi

$$r = 1.25 \text{ km S of dome}$$

$$d = 13.50^{10.17}_{17.33} \text{ km}$$

$$\Delta V = 0.0194^{0.0092}_{0.0340} \text{ km}^3$$

# 2009 Redoubt Source Evolution



# 2009 Redoubt Source Evolution

## Main Results:

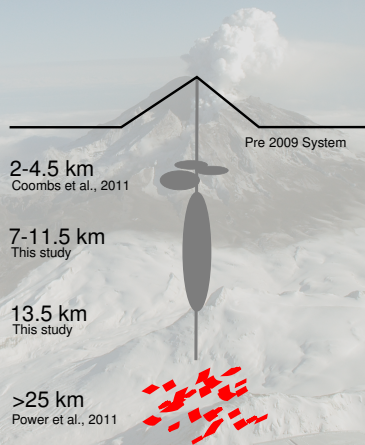
- pre-eruptive intrusion preceded seismic precursors
- dynamic change of source over weeks
- suggested process:



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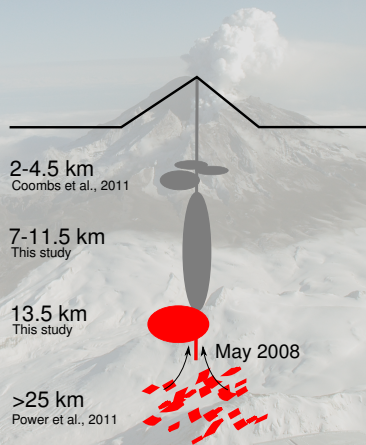
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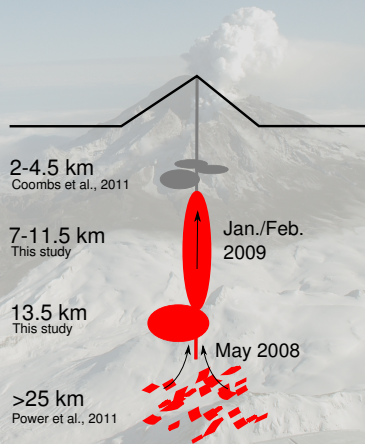
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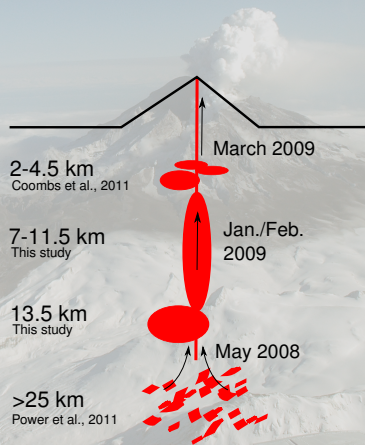
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