GEOS F493 / F693
Geodetic Methods and Modeling

– Lecture 12: Modeling - Surface Loading –

Ronni Grapenthin
rgrapenthin@alaska.edu
Elvey 413B
(907) 474-7286

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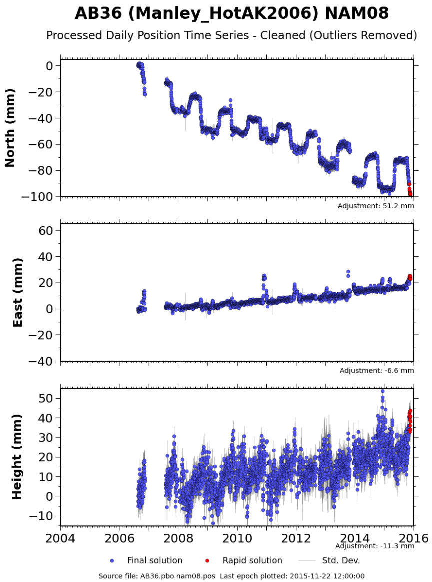
GUESS THE PROCESS!



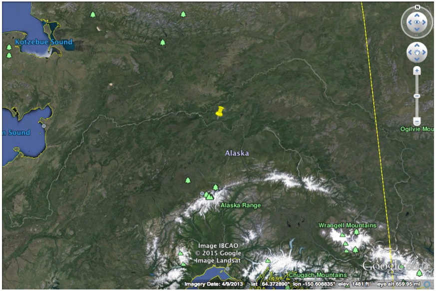
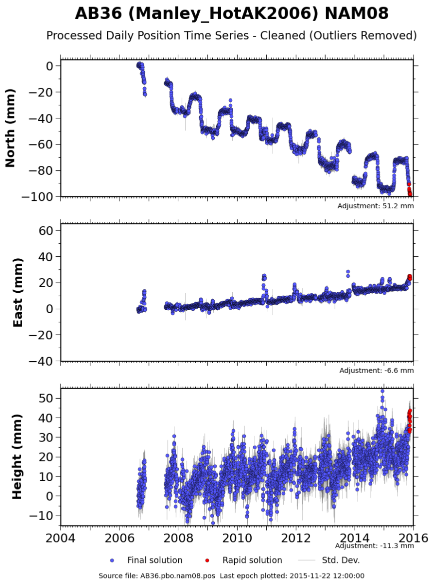


Figure 1. Photo of station AB36, taken in July 2011. Camera is pointing to the north. Rock outcrop is oriented east-west.

GUESS THE PROCESS!



UNAVCO

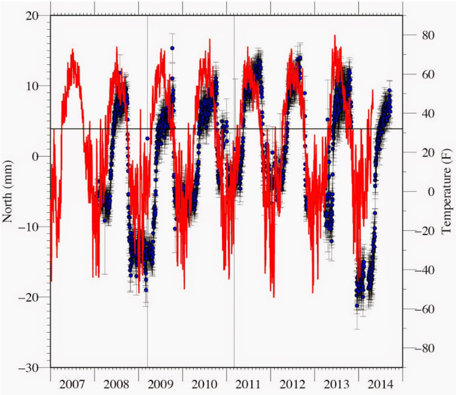
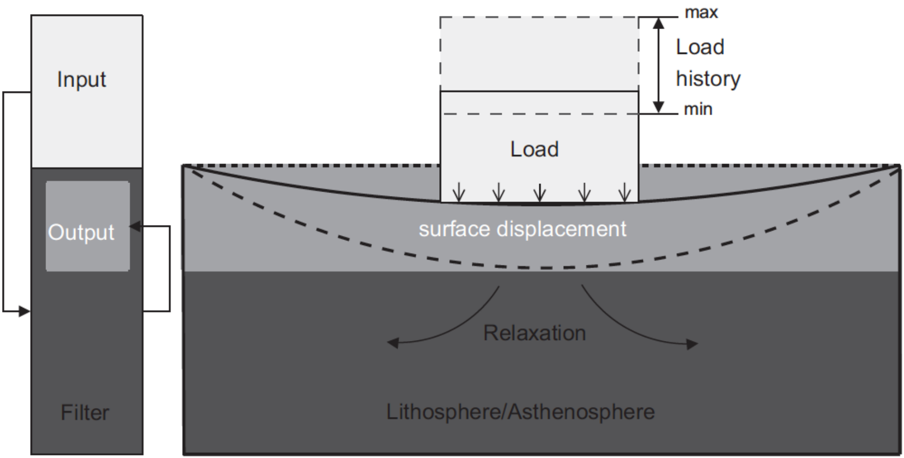


Figure 3. Detrended north motion (circles) at AB36 and daily mean tempetature at Manley Hot Springs Airport (red lines). Horizontal line marks freezing temperature (32°F).

Loading Deformation



Grapenthin, 2014, Comp. & Geosc.

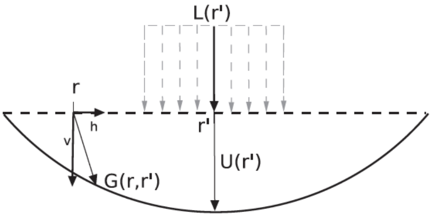
Computing Loading Deformation

1. Green's function method (unit impulse responses)
   - compute deformation induced by point load, specific shape, assume unit magnitude
   - convolve with actual spatial load (e.g., grid of point loads) with respective Green's function
2. Love's loading theory (Spherical Harmonics)
   - represent load in terms of spherical harmonic functions
   - compute deformation with Love Numbers that depend on Earth model

In practice Green's functions often come from Love's theory.

E.g., Spada, G., 2008. *ALMA, a Fortran program for computing the visco-elastic Love numbers of a spherically symmetric planet*, Comput. and Geosci., v. 34 (6), 667-687, doi: 0.1016/j.cageo.2007.12.001.

Green's Functions

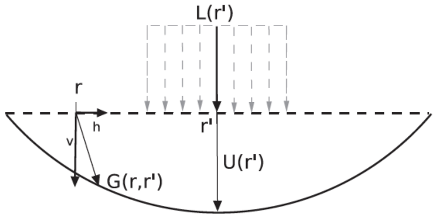


Grapenthin, 2014, Comp. & Geosc.

$$
\begin{aligned}
U(r) &= \int_A G(r - r')L(r')dS \\
U &= G ** L
\end{aligned}
$$

- integrate over area A in differential area steps dS
- "$**$" is 2D convolution operation

Green's Functions



Grapenthin, 2014, Comp. & Geosc.

$$U(r) = \int_A G(r - r')L(r')dS$$
$$U = G ** L$$

- integrate over area A in differential area steps dS
- "$**$" is 2D convolution operation

Green' Functions, e.g.:

- *Farrell 1972*: elastic spherical Earth (tabulated GF)
- *Pinel et al. 2007*: elastic, thick plate over visco-elastic fluid, flat Earth (GF)
- *Comer, 1983*: thin plate approximation (vertical normal stress=0)

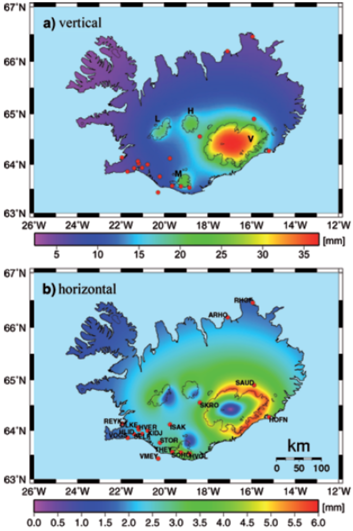
Green's Functions - generic Formulation

Grapenthin, 2014, Comp & Geosc.:

$$U_t = \sum_{i=1}^{n} [G_i * *L] \cdot (R_i * H_i)_t$$

- $*$ is convolution operator!
- n total number of superpositions of different Green's functions
- R relaxation function (e.g., exponential decay, delta function), wrt to Green's function
- H load history
- notation allows very generalized implementation of this problem (*CrusDe*).

Loading Deformation: Examples



3.1. Spatial Load Response

[8] Green's functions are a mathematical tool for solving linear differential equations which are derived for each specific problem. In order to get an estimate of the Earth's elastic response to a load, we consider an elastic halfspace and convolve Green's functions with the load as explained by *Pinel et al.* [2006]. Displacements are given as:

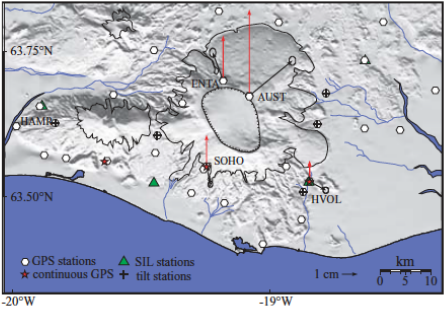
$$U_z(\vec{r}) = \int_R \frac{g}{\pi} \frac{(1-\nu^2)}{E} \frac{1}{|\vec{r}-\vec{r'}|} \rho(\vec{r'}) h(\vec{r'}) d\vec{r'} \qquad (1)$$

$$U_r(\vec{r}) = \int_R -\frac{g}{2\pi} \frac{(1+\nu)(1-2\nu)}{E} \frac{1}{|\vec{r}-\vec{r'}|} \rho(\vec{r'}) h(\vec{r'}) d\vec{r'} \qquad (2)$$

where U_z and U_r are, respectively, vertical and horizontal displacement at a point $\vec{r}$ (cylindrical coordinates). The elastic parameters characterizing the crust are the Poisson's ratio, ν, and effective Young's modulus, E; g is the acceleration due to gravity. The load's characteristics are the density, ρ, and the thickness, h, within the area R. An advantage of Equations 1 and 2 over traditional disk models comes with their allowance to apply arbitrarily shaped loads in a simple way. At each point $\vec{r'}$ in area R the load's height at this point, $h(\vec{r'})$, can be defined freely. Displacement at point $\vec{r}$ depends on the $|\vec{r} - \vec{r'}|$ distance.

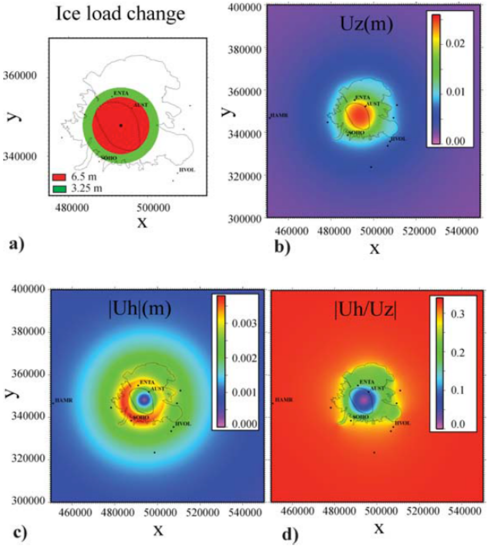
Grapenthin et al., 2006, GRL

Loading Deformation: Examples



Pinel et al., 2007, GJI

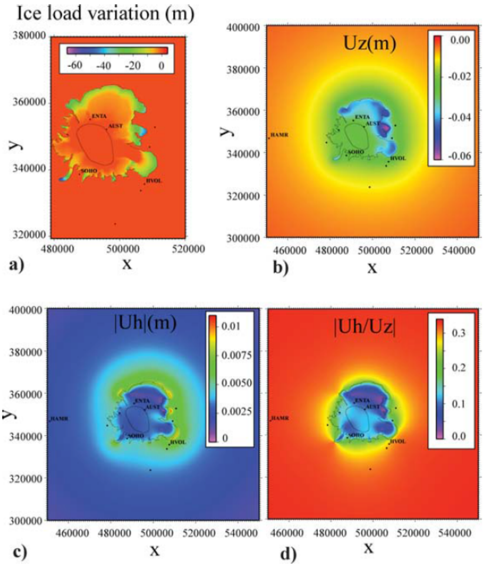
2 Disk Loads – elastic response:



Pinel et al., 2007, GJI

Loading Deformation: Examples

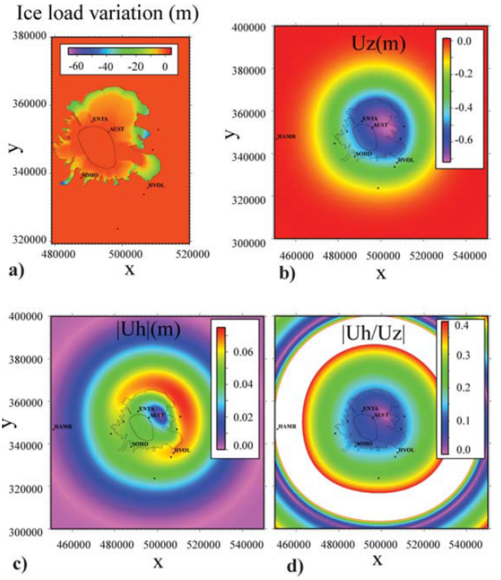
Irregular Load outline – elastic response:



Pinel et al., 2007, GJI

Loading Deformation: Examples

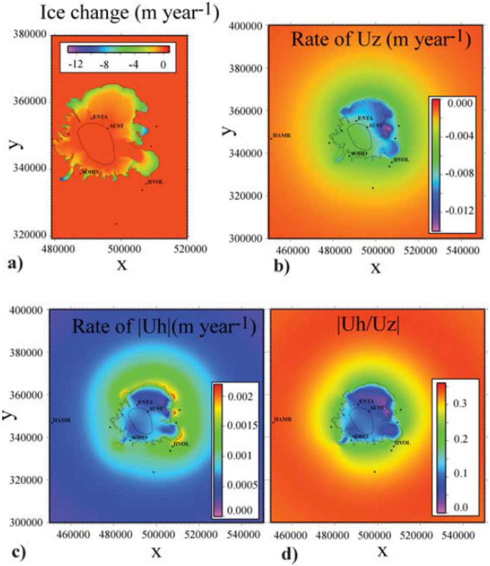
Irregular Load outline – final relaxed:



Pinel et al., 2007, GJI

Loading Deformation: Examples

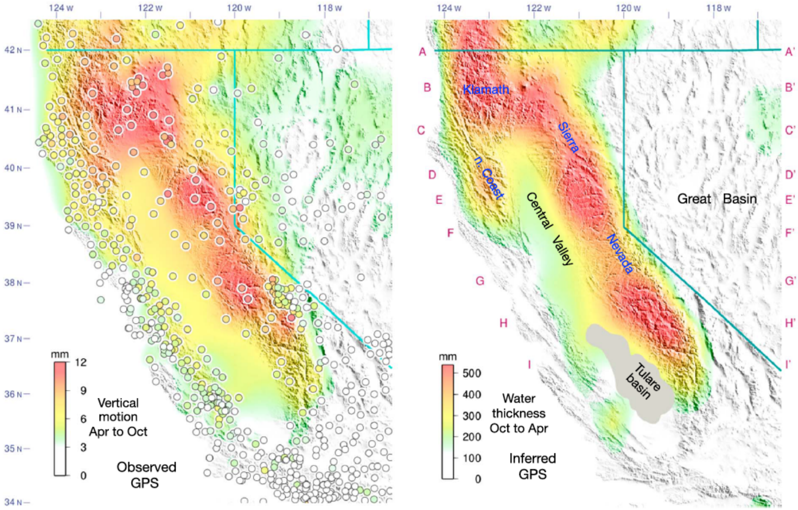
Irregular Load outline – annual rate given 115 yrs of ice thinning:



Pinel et al., 2007, GJI

Loading Deformation: Examples

Formal inversion of vertical GPS for seasonal loading:



Argus et al., 2014, GRL