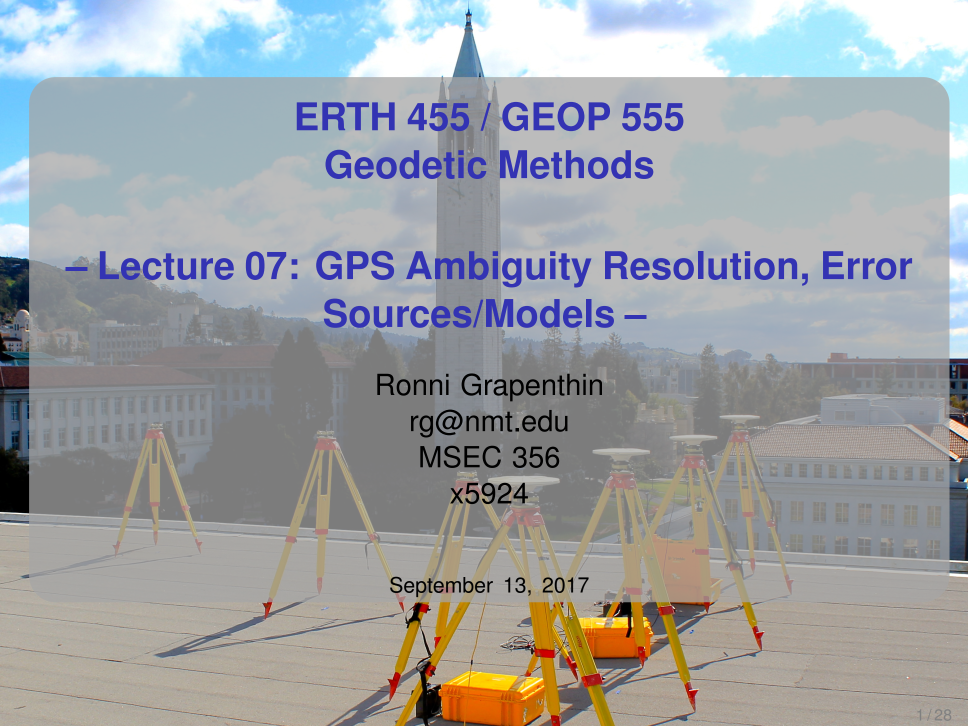


ERTH 455 / GEOP 555
Geodetic Methods

– Lecture 07: GPS Ambiguity Resolution, Error Sources/Models –

Ronni Grapenthin
rg@nmt.edu
MSEC 356
x5924

September 13, 2017

Geometry Issues

- Position estimate depends on quality (ϵ) and geometry (θ) of range measurement

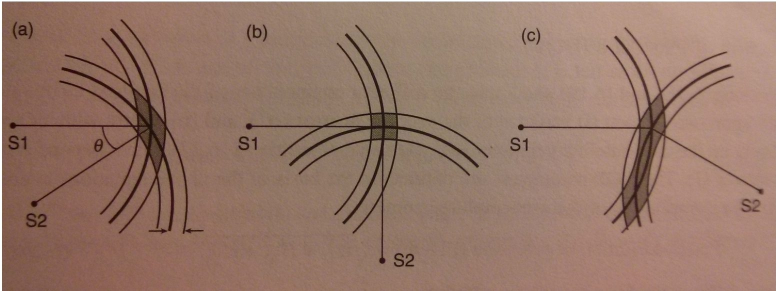


Figure 6.2 A simple 2-D example of position estimation with range measurements. The quality of the position estimate depends upon both the quality (ε) and geometry (θ) of the range measurements. The shaded region represents the uncertainty in the position estimate.

Misra and Enge, 2011, GPS–Signals, Measurements, and Performance

Carrier Phase Measurement

$$\phi = \frac{1}{\lambda} * (r + I + T) + f * (\delta t_u - \delta t^s) + N + \epsilon_\phi$$

(units of cycles) where

- λ, f - carrier wavelength, frequency
- r - geometric range
- I, T - ionospheric, tropospheric propagation errors (path delays)
- $\delta t_u, \delta t^s$ - receiver, satellite clock biases
- N - phase ambiguity
- ϵ_ϕ - error term (phase)

compare to code measurement eqn (units of distance):

$$\rho = r + I + T + c * (\delta t_u - \delta t^s) + \epsilon_\rho$$

Code tracking: unambiguous (long!)
$\sigma(\epsilon_\rho) \approx 0.5\,\text{m}$
$\sigma(\epsilon_\phi) \approx 0.025\,\text{cycle}\ (5\,\text{mm})$

Integer Ambiguity Resolution 1/5

- uncertainty in integer estimation depends on carrier wavelength
- increase wavelength –> decrease uncertainty: create *wide lane measurement*:

$$\begin{aligned}\phi_{L12} &= \phi_{L1} - \phi_{L2} \\ &= r(f_{L1} - f_{L2})/c + (N_{L1} - N_{L2}) + \epsilon_{\phi_{L12}} \\ &= r/\lambda_{L12} + N_{L12} + \epsilon_{\phi_{L12}}\end{aligned}$$

where $\lambda_{L12} = c/(f_{L1} - f_{L2}) = 0.862\,\text{m}$
$f_{L12} = f_{L1} - f_{L2} = 347.82\,\text{MHz}$
N_{L12} is integer ambiguity

Integer Ambiguity Resolution 2/5

Using

$$\rho_{L1} = r + \epsilon_{\rho_{L1}}$$

we can form estimate of N_{L12} as:

$$N_{L12} \approx \left[\phi_{L12} - \frac{\rho_{L1}}{\lambda_{L12}} \right]_{roundoff}$$

Here, $\sigma(N_{L12}) \approx 1.2$ cycles;
compared to $\sigma(N_{L1}) \approx 5$ cycles

Integer Ambiguity Resolution 3/5

- wide lane measurements much noisier than L1,L2 measurements
- *narrow lane combination* $\phi_{Ln} = \phi_{L1} + \phi_{L2}$ less noisy
- though harder to resolve ambiguities with narrow lane
- position estimates would be more precise

Integer Ambiguity Resolution (one at a time) 4/5

With correct N_{L12} can determine N_{L1}, N_{L2}. Measurement eqs:

$$\begin{aligned} \phi_{L1} &= r/\lambda_{L1} + N_{L1} + \epsilon_{\phi_{L1}} \\ \phi_{L2} &= r/\lambda_{L2} + N_{L2} + \epsilon_{\phi_{L2}} \end{aligned}$$

after solving both for r and equating, we get:

$$N_{L1} - \frac{\lambda_{L2}}{\lambda_{L1}} N_{L2} = \phi_{L1} - \frac{\lambda_{L2}}{\lambda_{L1}} \phi_{L2} + \epsilon$$

Integer Ambiguity Resolution (one at a time) 4/5

With correct N_{L12} can determine N_{L1}, N_{L2}. Measurement eqs:

$$\begin{aligned} \phi_{L1} &= r/\lambda_{L1} + N_{L1} + \epsilon_{\phi_{L1}} \\ \phi_{L2} &= r/\lambda_{L2} + N_{L2} + \epsilon_{\phi_{L2}} \end{aligned}$$

after solving both for r and equating, we get:

$$N_{L1} - \frac{\lambda_{L2}}{\lambda_{L1}} N_{L2} = \phi_{L1} - \frac{\lambda_{L2}}{\lambda_{L1}} \phi_{L2} + \epsilon$$

We have

$$\begin{aligned} N_{L1} - N_{L2} &= N_{L12} \\ N_{L2} &= N_{L1} - N_{L12} \end{aligned}$$

Integer Ambiguity Resolution (one at a time) 4/5

With correct N_{L12} can determine N_{L1}, N_{L2}. Measurement eqs:

$$\begin{aligned}\phi_{L1} &= r/\lambda_{L1} + N_{L1} + \epsilon_{\phi_{L1}} \\ \phi_{L2} &= r/\lambda_{L2} + N_{L2} + \epsilon_{\phi_{L2}}\end{aligned}$$

after solving both for r and equating, we get:

$$N_{L1} - \frac{\lambda_{L2}}{\lambda_{L1}} N_{L2} = \phi_{L1} - \frac{\lambda_{L2}}{\lambda_{L1}} \phi_{L2} + \epsilon$$

We have

$$\begin{aligned}N_{L1} - N_{L2} &= N_{L12} \\ N_{L2} &= N_{L1} - N_{L12}\end{aligned}$$

So, we can solve for N_{L1}, N_{L2}:

$$N_{L1} = \left[\frac{\lambda_{L2}}{\lambda_{L1}} - 1\right]^{-1} \left[\frac{\lambda_{L2}}{\lambda_{L1}} N_{L12} - \phi_{L1} + \frac{\lambda_{L2}}{\lambda_{L1}} \phi_{L2}\right]$$

Integer Ambiguity Resolution (one at a time) 4/5

With correct N_{L12} can determine N_{L1}, N_{L2}. Measurement eqs:

$$\begin{aligned} \phi_{L1} &= r/\lambda_{L1} + N_{L1} + \epsilon_{\phi_{L1}} \\ \phi_{L2} &= r/\lambda_{L2} + N_{L2} + \epsilon_{\phi_{L2}} \end{aligned}$$

after solving both for r and equating, we get:

$$N_{L1} - \frac{\lambda_{L2}}{\lambda_{L1}} N_{L2} = \phi_{L1} - \frac{\lambda_{L2}}{\lambda_{L1}} \phi_{L2} + \epsilon$$

We have

$$\begin{aligned} N_{L1} - N_{L2} &= N_{L12} \\ N_{L2} &= N_{L1} - N_{L12} \end{aligned}$$

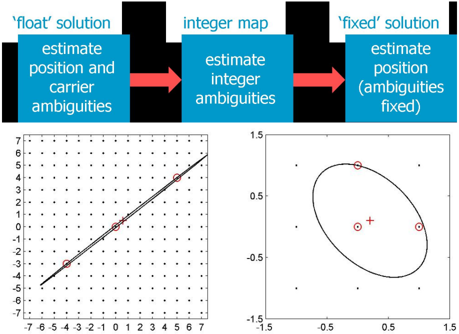
So, we can solve for N_{L1}, N_{L2}:

$$N_{L1} = \left[\frac{\lambda_{L2}}{\lambda_{L1}} - 1\right]^{-1} \left[\frac{\lambda_{L2}}{\lambda_{L1}} N_{L12} - \phi_{L1} + \frac{\lambda_{L2}}{\lambda_{L1}} \phi_{L2}\right]$$

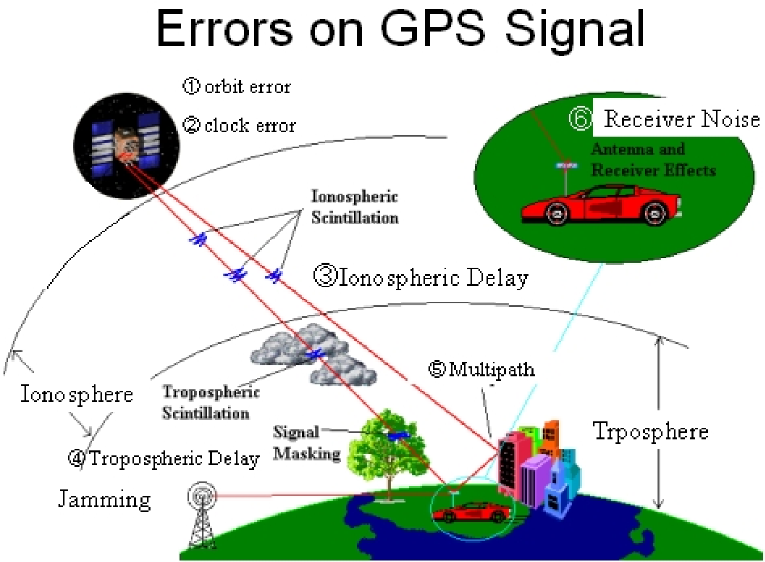
Uncertainty $\sigma(N_{L1}) \approx 6\sigma(\epsilon_{\phi_{L_1}})$; data quality determines success.

Integer Ambiguity Resolution (as a set) 5/5

- 1) discard integer nature of ambiguities and find least squares 'float solution'
- 2) map to integer (decorrelate error elipse)
- 3) 'fixed solution': estimate position (other parameters) w/ integer ambiguities



http://www.citg.tudelft.nl/en/about-faculty/departments/geoscience-and-remote-sensing/research-themes/gps/lambda-method/



http://www.blackboxcamera.com

Ionospheric Delay 1/5

carrier phase (unit of cycles):

$$\phi = \frac{1}{\lambda} * (r + \mathbf{I} + T) + f * (\delta t_u - \delta t^s) + N + \epsilon_\phi$$

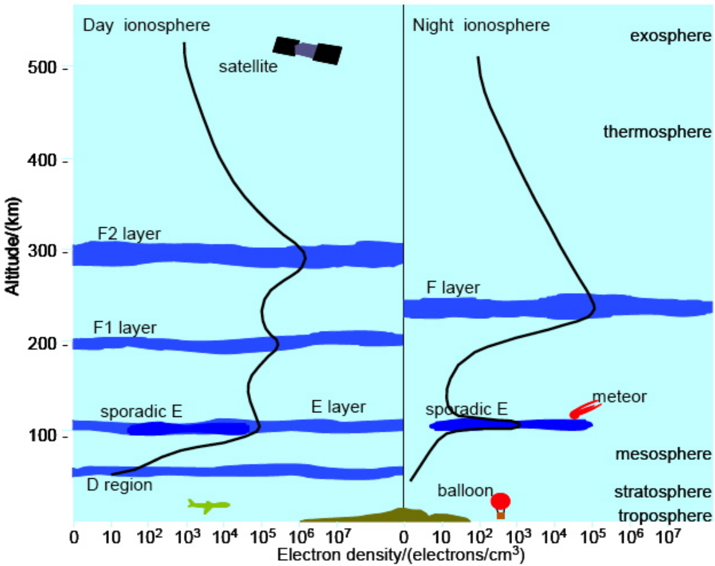
code measurement eqn (units of distance):

$$\rho = r + \mathbf{I} + T + c * (\delta t_u - \delta t^s) + \epsilon_\rho$$

Ionospheric Delay 2/5

- $\approx$ 50-1000 km above Earth
- ionized gases: free electrons and ions, sun's radiation/activity drives state
- daily cycle with peak electron density at about 2 pm local time
- electron density 1-2 orders of magnitude difference between night/day
- changes w/ seasons, 11-year solar cycle, other short term anomalies (tsunamis), solar flares
- dispersive for GPS frequencies (different frequencies - different effective velocity)

Ionospheric Delay 3/5



http://www.amateur-radio-wiki.net

Ionospheric Delay 4/5

Total electron content (TEC): number of electrons $n_e(l)$ in tube of 1 m^2 connecting satellite and receiver:

$$TEC = \int_R^S n_e(l) dl \text{ (TECU: TEC units)}$$

- *VTEC*: TEC in vertical direction, lowest TEC when satellite in zenith direction
- *VTEC* between 1-150 TECU
- region with highest ionospheric delay within $\pm 20^\circ$ of magnetic equator

Ionospheric Delay 5/5

Dual-frequency receivers allow (basically) elimination of ionosphere as source of error. Possible combinations for code range and carrier phases (derivation in Hofmann-Wellenhof et al.)

$$\begin{aligned}\rho_{L1,L2} &= \rho_{L1} - \frac{f_{L2}^2}{f_{L1}^2}\rho_{L2} \\ \phi_{L1,L2} &= \phi_{L1} - \frac{f_{L2}}{f_{L1}}\phi_{L2}\end{aligned}$$

other combinations possible.

Model exists to remove ionosphere from L1-only observations (*Klobuchar model*, [Klobuchar, 1996]).

Tropospheric Delay 1/4

carrier phase (unit of cycles):

$$\phi = \frac{1}{\lambda} * (r + I + \mathbf{T}) + f * (\delta t_u - \delta t^s) + N + \epsilon_\phi$$

code measurement eqn (units of distance):

$$\rho = r + I + \mathbf{T} + c * (\delta t_u - \delta t^s) + \epsilon_\rho$$

Tropospheric Delay 2/4

- troposphere non-dispersive for GPS frequencies (common delay for L1, L2 frequency)

Tropospheric Delay 2/4

- troposphere non-dispersive for GPS frequencies (common delay for L1, L2 frequency)
- about 2.5-25 m range lengthening dep. on elevation angle

Tropospheric Delay 2/4

- troposphere non-dispersive for GPS frequencies (common delay for L1, L2 frequency)
- about 2.5-25 m range lengthening dep. on elevation angle
- need models to correct

Tropospheric Delay 2/4

- troposphere non-dispersive for GPS frequencies (common delay for L1, L2 frequency)
- about 2.5-25 m range lengthening dep. on elevation angle
- need models to correct
- refractive index (n) is ratio of propagation speed of signal in vacuum (c) to speed in a medium (ν): $n = c/\nu$

Tropospheric Delay 2/4

- troposphere non-dispersive for GPS frequencies (common delay for L1, L2 frequency)
- about 2.5-25 m range lengthening dep. on elevation angle
- need models to correct
- refractive index (n) is ratio of propagation speed of signal in vacuum (c) to speed in a medium (ν): $n = c/\nu$
- refractivity $N' = (n - 1) \times 10^6$ is sum dry gas and water vapor refractivities: $N' = N'_d + N'_w$

Tropospheric Delay 2/4

- troposphere non-dispersive for GPS frequencies (common delay for L1, L2 frequency)
- about 2.5-25 m range lengthening dep. on elevation angle
- need models to correct
- refractive index (n) is ratio of propagation speed of signal in vacuum (c) to speed in a medium (ν): $n = c/\nu$
- refractivity $N' = (n - 1) \times 10^6$ is sum dry gas and water vapor refractivities: $N' = N'_d + N'_w$
- so $T = T_d + T_w = 10^-6 \int N'_d(l) + N'_w(l) dl$, T_d, T_w dry and wet tropospheric delay

Tropospheric Delay 2/4

- troposphere non-dispersive for GPS frequencies (common delay for L1, L2 frequency)
- about 2.5-25 m range lengthening dep. on elevation angle
- need models to correct
- refractive index (n) is ratio of propagation speed of signal in vacuum (c) to speed in a medium (ν): $n = c/\nu$
- refractivity $N' = (n - 1) \times 10^6$ is sum dry gas and water vapor refractivities: $N' = N'_d + N'_w$
- so $T = T_d + T_w = 10^-6 \int N'_d(l) + N'_w(l)dl$, T_d, T_w dry and wet tropospheric delay
- refractivity of air parcel depends on temperature, partial pressures of dry gases, water vapor

Tropospheric Delay 2/4

- troposphere non-dispersive for GPS frequencies (common delay for L1, L2 frequency)
- about 2.5-25 m range lengthening dep. on elevation angle
- need models to correct
- refractive index (n) is ratio of propagation speed of signal in vacuum (c) to speed in a medium (ν): $n = c/\nu$
- refractivity $N' = (n - 1) \times 10^6$ is sum dry gas and water vapor refractivities: $N' = N'_d + N'_w$
- so $T = T_d + T_w = 10^-6 \int N'_d(l) + N'_w(l)dl$, T_d, T_w dry and wet tropospheric delay
- refractivity of air parcel depends on temperature, partial pressures of dry gases, water vapor
- approximations: $N_d = 77.64P/T$ and $N_w = 3.73 \times 10^5 e/T^2$ with P total, e partial pressures (mB), T temperature in K.

Tropospheric Delay 3/4

- would be best to radiosonde atmosphere to get pressure, temperature, humidity profile: impractical

Tropospheric Delay 3/4

- would be best to radiosonde atmosphere to get pressure, temperature, humidity profile: impractical
- next best: metpack at stations for meteorological measurements

Tropospheric Delay 3/4

- would be best to radiosonde atmosphere to get pressure, temperature, humidity profile: impractical
- next best: metpack at stations for meteorological measurements
- practice: estimate tropospheric delay based on standard atmosphere given day of year and user location

Tropospheric Delay 3/4

- would be best to radiosonde atmosphere to get pressure, temperature, humidity profile: impractical
- next best: metpack at stations for meteorological measurements
- practice: estimate tropospheric delay based on standard atmosphere given day of year and user location
- 2 step process:
  - estimate zenith delay based on model $T_z = T_{z,d} + T_{z,w}$
  - mapping function to scale zenith delay as function of satellite elevation angle (θ) $T = T_{z,d} m_d(\theta) + T_{z,w} m_w(\theta)$

Tropospheric Delay 3/4

- would be best to radiosonde atmosphere to get pressure, temperature, humidity profile: impractical
- next best: metpack at stations for meteorological measurements
- practice: estimate tropospheric delay based on standard atmosphere given day of year and user location
- 2 step process:
  - estimate zenith delay based on model $T_z = T_{z,d} + T_{z,w}$
  - mapping function to scale zenith delay as function of satellite elevation angle (θ) $T = T_{z,d} m_d(\theta) + T_{z,w} m_w(\theta)$
- models: differ in assumptions regarding changes of temperature and water vapor with altitude

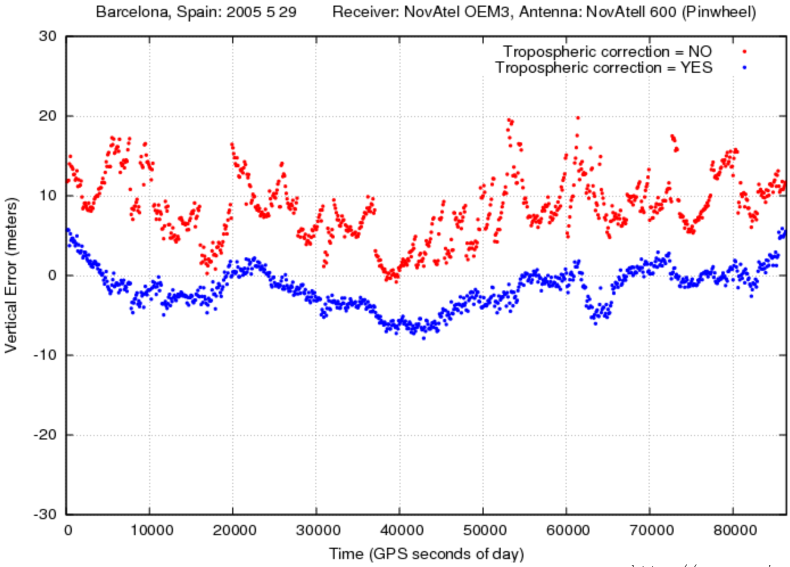
Tropospheric Delay 3/4

- would be best to radiosonde atmosphere to get pressure, temperature, humidity profile: impractical
- next best: metpack at stations for meteorological measurements
- practice: estimate tropospheric delay based on standard atmosphere given day of year and user location
- 2 step process:
  - estimate zenith delay based on model $T_z = T_{z,d} + T_{z,w}$
  - mapping function to scale zenith delay as function of satellite elevation angle (θ) $T = T_{z,d} m_d(\theta) + T_{z,w} m_w(\theta)$
- models: differ in assumptions regarding changes of temperature and water vapor with altitude
- mapping functions: vary in geometric complexity assumptions (simplest: $1/sin(\theta)$)

Tropospheric Delay 3/4

- would be best to radiosonde atmosphere to get pressure, temperature, humidity profile: impractical
- next best: metpack at stations for meteorological measurements
- practice: estimate tropospheric delay based on standard atmosphere given day of year and user location
- 2 step process:
  - estimate zenith delay based on model $T_z = T_{z,d} + T_{z,w}$
  - mapping function to scale zenith delay as function of satellite elevation angle (θ) $T = T_{z,d} m_d(\theta) + T_{z,w} m_w(\theta)$
- models: differ in assumptions regarding changes of temperature and water vapor with altitude
- mapping functions: vary in geometric complexity assumptions (simplest: $1/sin(\theta)$)
- lots of both exist

Tropospheric Delay 4/4



http://www.navipedia.net

Receiver Noise

carrier phase (unit of cycles):

$$\phi = \frac{1}{\lambda} * (r + I + T) + f * (\delta t_u - \delta t^s) + N + \epsilon_\phi$$

code measurement eqn (units of distance):

$$\rho = r + I + T + c * (\delta t_u - \delta t^s) + \epsilon_\rho$$

Receiver Noise

- RF radiation sensed by antenna (interference)
- noise introduced by antenna, amplifiers, cables (!), receiver, signal quantization!
- absence of interference: rcx sees waveform = GPS + random noise
- fine structure of signal can be masked by noise (esp. low SNR)
- error varies w/ signal strength, which depends on satellite elev angle

Multipath

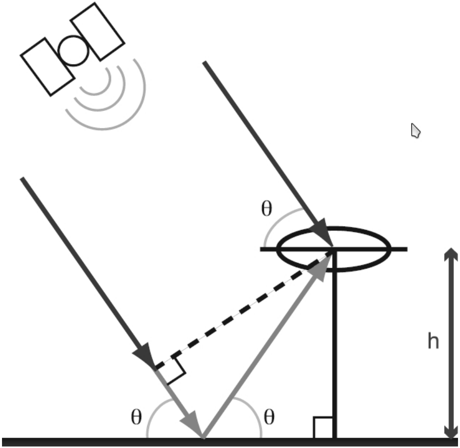
carrier phase (unit of cycles):

$$\phi = \frac{1}{\lambda} * (r + I + T) + f * (\delta t_u - \delta t^s) + N + \mathbf{MP} + \epsilon_\phi$$

code measurement eqn (units of distance):

$$\rho = r + I + T + c * (\delta t_u - \delta t^s) + \mathbf{MP} + \epsilon_\rho$$

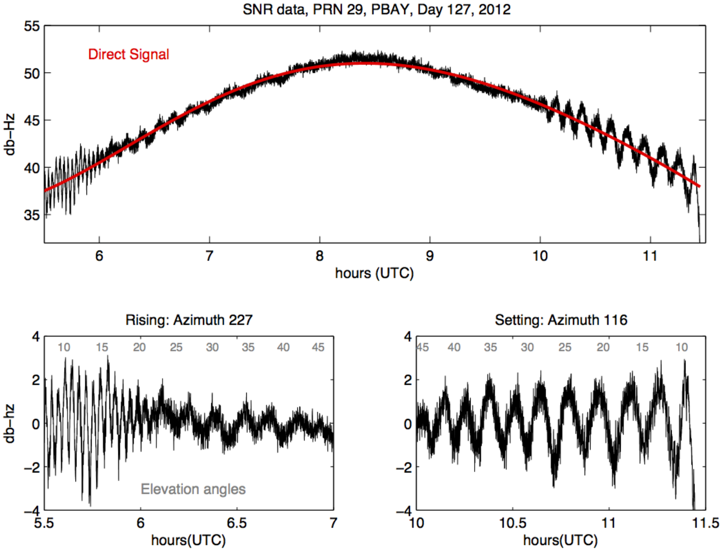
Multipath



Larson et al. (2007)

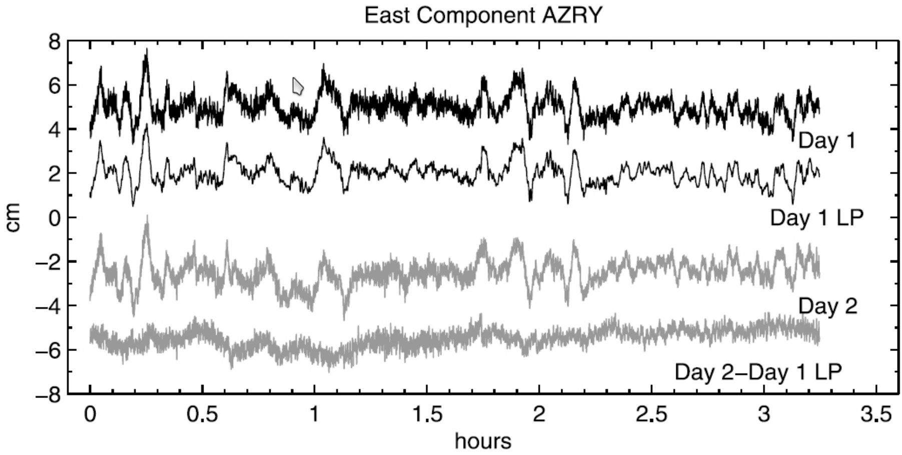
- best seen in subdaily solutions
- signal reaches antenna via direct and indirect paths
- reflected signal delayed, weaker
- mitigation: antenna design, receiver algorithms
- code and phase measurement are sum of received signals
- pseudorange: 1-5 m error
- phase: 1-5 cm error (no worse than 1/4 cycle)

Multipath in SNR data



http://xenon.colorado.edu/spotlight/index.php?product=spotlight&station=PBAY

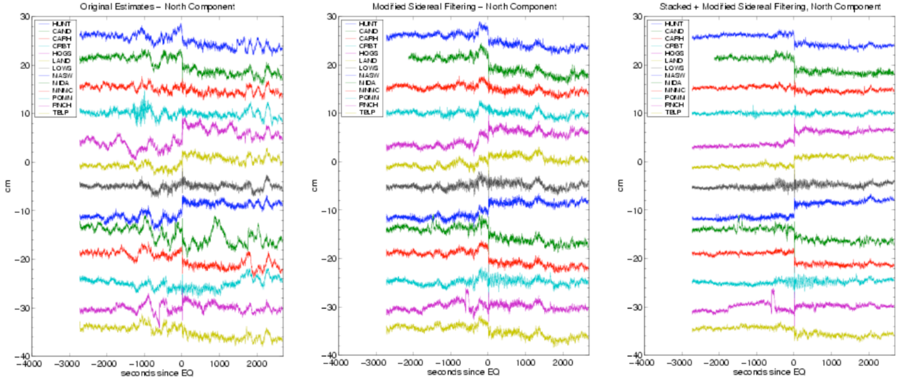
Eliminating Multi-Path through Sidereal Filtering



Larson et al. (2007)

Eliminating Multi-Path through Sidereal Filtering

Parkfield earthquake



Andria Bilich, University of Colorado

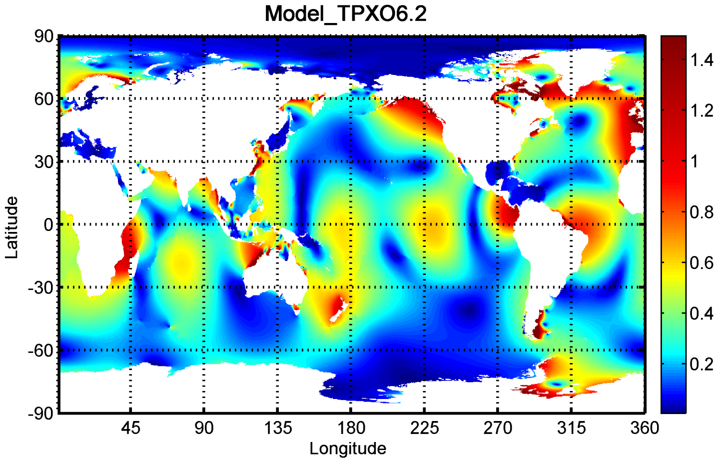
Ocean Tidal Loading

- solid earth responds to changing load due to ocean tides
- large near coast (with large tidal range, depends on coastline)
- need good tidal models for removal

e.g., TPXO6:

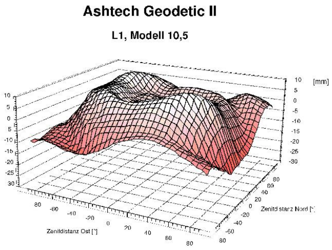
- eight primary constituents M2, S2, N2, K2, K1, O1, P1, Q1
- two long period Mf,Mm constituents
- three non-linear M4, MS4, MN4 harmonic constituents
- on 1/4 degree resolution full global grid (for versions 6.* and later).

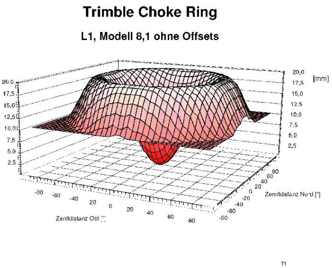
Ocean Tidal Loading



Map of M2 sea surface height amplitude (m) from TPXO6.2
https://www.esr.org/polar_tide_models/Model_TPXO62.html

Antenna Phase Center Models





- imaginary point in space that we measure distances to
- different for every type of antenna
- ideally point in space, but depends on azimuth and elevation of signal
- models assume azimuthal independence, fit elevation

`http://www.geopp.de/gnpcv/`