ERTH 491-01 / GEOP 572-02
Geodetic Methods

– Lecture 07: GPS Carrier Phase –

Ronni Grapenthin
rg@nmt.edu
MSEC 356
x5924

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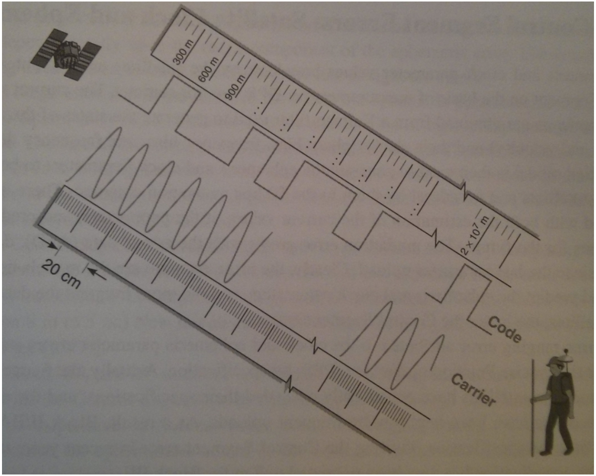
Measurement Models

- Code Phase Measurement (last week)
- Carrier Phase Measurement (today!)



Misra and Enge, 2011, GPS–Signals, Measurements, and Performance

Carrier Phase Measurement

- also: carrier beat phase measurement
- difference between phases of receiver generated carrier signal and carrier received from satellite
- is indirect and ambiguous measurement of signal transit time
- phase at time *t*:

$$\phi(t) = \phi_u(t) - \phi^s(t - \tau) + N$$

-
    - $\phi_u(t)$ phase of rcx generated signal
    - $\phi^S(t - \tau)$ phase of satellite signal received at t (sent at $t - \tau$)
    - τ: still transit time
    - N: integer ambiguity, must be estimated: *integer ambiguity resolution*

Carrier Phase Measurement

use (f is carrier frequency):

$$\phi^S(t-\tau) = \phi^S(t) - f * \tau$$

to get

$$\begin{aligned} \phi(t) &= \phi_u(t) - \phi^s(t-\tau) + N \\ &= \phi_u(t) - \phi^S(t) + f * \tau + N \\ &= f * \tau + N \\ &= \frac{r}{\lambda} + N \end{aligned}$$

where r is geometric range, λ is carrier wavelength

Carrier Phase Measurement

$$\phi = \frac{1}{\lambda} * (r + I + T) + f * (\delta t_u - \delta t^s) + N + \epsilon_\phi$$

(units of cycles) where

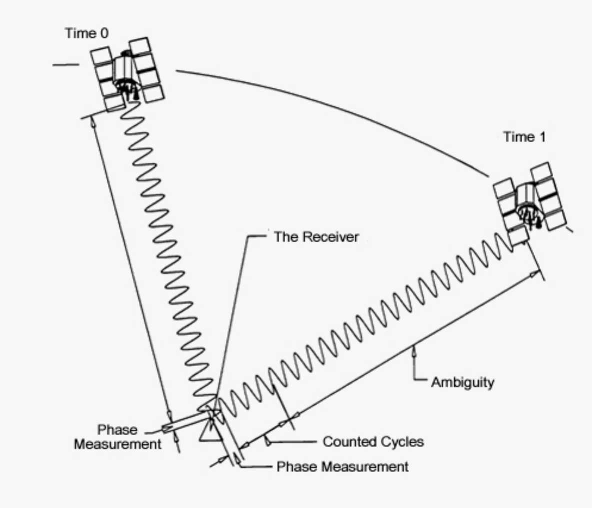
- λ, f - carrier wavelength, frequency
- r - geometric range
- I, T - ionospheric, tropospheric propagation errors (path delays)
- $\delta t_u, \delta t^s$ - receiver, satellite clock biases
- N - phase ambiguity
- ϵ_ϕ - error term (phase)

compare to code measurement eqn (units of distance):

$$\rho = r + I + T + c * (\delta t_u - \delta t^s) + \epsilon_\rho$$

Code tracking: unambiguous (long!)
$\sigma(\epsilon_\rho) \approx 0.5\,\mathrm{m}$
$\sigma(\epsilon_\phi) \approx 0.025\,\mathrm{cycle}\ (5\,\mathrm{mm})$

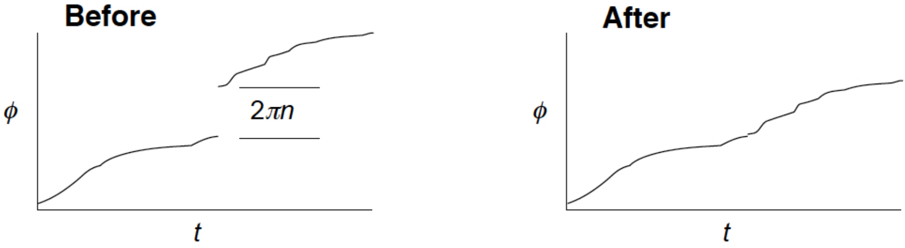
Phase Ambiguity



http://nptel.ac.in/courses/105104100/lectureB_8/B_8_4carrier.htm

Cycle Slip

- receiver has to track phase continuously
- loss of lock (tree, etc): cycle slip – integer number of cycles jump in phase data
- must be fixed during analysis (software, several strategies; sometimes manually)

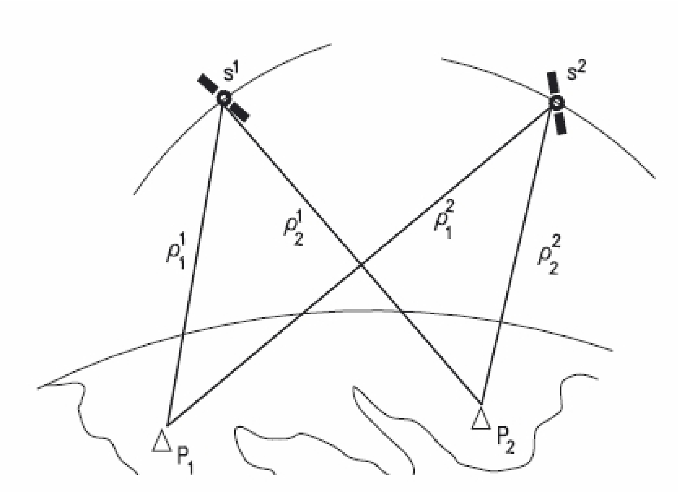


courtesy: *Jeff Freymueller*

Elimination of “Nuisance” Parameters

- difference multiple satellite and receiver data to eliminate clock biases
- “single difference” between 2 receivers and 1 satellite: eliminates satellite clock
- “single difference” between 1 receiver and 2 satellites: eliminates receiver clock
- “double difference” between those differences removes both clocks
- BUT: you estimate baseline vector between receivers rather than their positions!
- no linearly dependent observations, careful choosing (by software)
- some estimate clock errors intead

Single + Double Difference



http://www.fig.net/resources/publications/figpub/pub49/figpub49.asp

Single Difference

Carrier phase measurement from satellite k at receiver u:

$$\phi_u^{(k)} = \frac{1}{\lambda} * (r_u^{(k)} + I_u^{(k)} + T_u^{(k)}) + f * (\delta t_u - \delta \mathbf{t}^{(\mathbf{k})}) + N_u^{(k)} + \epsilon_{\phi,u}^{(k)}$$

Single Difference

Carrier phase measurement from satellite k at receiver u:

$$\phi_u^{(k)} = \frac{1}{\lambda} * (r_u^{(k)} + I_u^{(k)} + T_u^{(k)}) + f * (\delta t_u - \delta \mathbf{t}^{(\mathbf{k})}) + N_u^{(k)} + \epsilon_{\phi,u}^{(k)}$$

Carrier phase measurement from satellite k at receiver r:

$$\phi_r^{(k)} = \frac{1}{\lambda} * (r_r^{(k)} + I_r^{(k)} + T_r^{(k)}) + f * (\delta t_r - \delta \mathbf{t}^{(\mathbf{k})}) + N_r^{(k)} + \epsilon_{\phi,r}^{(k)}$$

Single Difference

Carrier phase measurement from satellite k at receiver u:

$$\phi_u^{(k)} = \frac{1}{\lambda} * (r_u^{(k)} + I_u^{(k)} + T_u^{(k)}) + f * (\delta t_u - \delta \mathbf{t}^{(\mathbf{k})}) + N_u^{(k)} + \epsilon_{\phi,u}^{(k)}$$

Carrier phase measurement from satellite k at receiver r:

$$\phi_r^{(k)} = \frac{1}{\lambda} * (r_r^{(k)} + I_r^{(k)} + T_r^{(k)}) + f * (\delta t_r - \delta \mathbf{t}^{(\mathbf{k})}) + N_r^{(k)} + \epsilon_{\phi,r}^{(k)}$$

receiver single difference:

$$\begin{aligned} \phi_{ur}^{(k)} &= \phi_u^{(k)} - \phi_r^{(k)} \\ &= \frac{1}{\lambda} * (r_{ur}^{(k)} + I_{ur}^{(k)} + T_{ur}^{(k)}) + f * \delta t_{ur} + N_{ur}^{(k)} + \epsilon_{\phi,ur}^{(k)} \end{aligned}$$

Single Difference

Carrier phase measurement from satellite k at receiver u:

$$\phi_u^{(k)} = \frac{1}{\lambda} * (r_u^{(k)} + I_u^{(k)} + T_u^{(k)}) + f * (\delta t_u - \delta \mathbf{t}^{(\mathbf{k})}) + N_u^{(k)} + \epsilon_{\phi,u}^{(k)}$$

Carrier phase measurement from satellite k at receiver r:

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receiver single difference:

$$\begin{aligned} \phi_{ur}^{(k)} &= \phi_u^{(k)} - \phi_r^{(k)} \\ &= \frac{1}{\lambda} * (r_{ur}^{(k)} + I_{ur}^{(k)} + T_{ur}^{(k)}) + f * \delta t_{ur} + N_{ur}^{(k)} + \epsilon_{\phi,ur}^{(k)} \end{aligned}$$

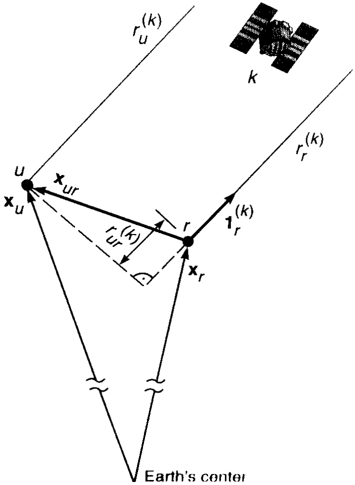
"short" baseline (ionosphere, troposphere errors small)

$$\phi_{ur}^{(k)} = \frac{r_{ur}^{(k)}}{\lambda} + f * \delta t_{ur} + N_{ur}^{(k)} + \epsilon_{\phi,ur}^{(k)}$$

want to estimate $\mathbf{x_{ur}} = \mathbf{x_u} - \mathbf{x_r}$ hidden in range difference (short baselines):

$$r_{ur}^{(k)} = r_u^{(k)} - r_r^{(k)} = -\mathbf{1_r^{(k)}} \mathbf{x_{ur}}$$

$\mathbf{1_r^{(k)}}$ is unit vector pointing from receiver *r* to satellite *k* (different treatment for longer baselines)



Misra and Enge, 2011, GPS–Signals, Measurements, and Performance

Single Difference

Single differences for all K satellites in view:

$$\phi_{ur} = \frac{1}{\lambda} \begin{bmatrix} (-\mathbf{1_r^{(1)}})^T \\ (-\mathbf{1_r^{(2)}})^T \\ \vdots \\ (-\mathbf{1_r^{(K)}})^T \end{bmatrix} \mathbf{x_{ur}} + f * \delta t_{ur} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} + \begin{bmatrix} N_{ur}^{(1)} \\ N_{ur}^{(2)} \\ \vdots \\ N_{ur}^{(K)} \end{bmatrix} + \epsilon_{\phi,ur}$$

Single Difference

Single differences for all K satellites in view:

$$\phi_{ur} = \frac{1}{\lambda} \begin{bmatrix} (-\mathbf{1}_{\mathbf{r}}^{(\mathbf{1})})^T \\ (-\mathbf{1}_{\mathbf{r}}^{(\mathbf{2})})^T \\ \vdots \\ (-\mathbf{1}_{\mathbf{r}}^{(\mathbf{K})})^T \end{bmatrix} \mathbf{x}_{\mathbf{ur}} + f * \delta t_{ur} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} + \begin{bmatrix} N_{ur}^{(1)} \\ N_{ur}^{(2)} \\ \vdots \\ N_{ur}^{(K)} \end{bmatrix} + \epsilon_{\phi,ur}$$

Can be rearranged to:

$$\phi_{ur} = \begin{bmatrix} (-\mathbf{1}_{\mathbf{r}}^{(\mathbf{1})})^T 1 \\ (-\mathbf{1}_{\mathbf{r}}^{(\mathbf{2})})^T 1 \\ \vdots\vdots \\ (-\mathbf{1}_{\mathbf{r}}^{(\mathbf{K})})^T 1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_{\mathbf{ur}} \\ b_{ur} + \lambda N_{ur}^{(1)} \end{bmatrix} + \begin{bmatrix} 0 \\ \lambda(N_{ur}^{(2)} - N_{ur}^{(1)}) \\ \vdots \\ \lambda(N_{ur}^{(K)} - N_{ur}^{(1)}) \end{bmatrix} + \epsilon_{\phi,ur}$$

where $b_{ur} = c\delta t_{ur}$ is receiver clock bias

Double Difference

Form single differences for receivers u, r and satellite l

$$\begin{aligned}\phi_{ur}^{(l)} &= \phi_u^{(l)} - \phi_r^{(l)} \\ &= \frac{r_{ur}^{(l)}}{\lambda} + \mathbf{f} * \delta \mathbf{t_{ur}} + N_{ur}^{(l)} + \epsilon_{\phi,ur}^{(l)}\end{aligned}$$

Double Difference

Form single differences for receivers u, r and satellite l

$$\begin{aligned}
\phi_{ur}^{(l)} &= \phi_{u}^{(l)} - \phi_{r}^{(l)} \\
&= \frac{r_{ur}^{(l)}}{\lambda} + \mathbf{f} * \delta \mathbf{t_{ur}} + N_{ur}^{(l)} + \epsilon_{\phi,ur}^{(l)}
\end{aligned}$$

Form double difference:

$$\begin{aligned}
\phi_{ur}^{(kl)} &= \phi_{ur}^{(k)} - \phi_{ur}^{(l)} \\
&= (\phi_{u}^{(k)} - \phi_{r}^{(k)}) - (\phi_{u}^{(l)} - \phi_{r}^{(l)}) \\
&= \frac{r_{ur}^{(kl)}}{\lambda} + N_{ur}^{(kl)} + \epsilon_{\phi,ur}^{(kl)}
\end{aligned}$$

Double Difference

relate range double difference term to relative position vector $\mathbf{x_{ur}}$:

$$\begin{aligned} r_{ur}^{(kl)} &= (r_u^{(k)} - r_r^{(k)}) - (r_u^{(l)} - r_r^{(l)}) \\ &= -(\mathbf{1_r^{(k)}} - \mathbf{1_r^{(l)}})\mathbf{x_{ur}} \end{aligned}$$

Double Difference

relate range double difference term to relative position vector $\mathbf{x_{ur}}$:

$$\begin{aligned} r_{ur}^{(kl)} &= (r_u^{(k)} - r_r^{(k)}) - (r_u^{(l)} - r_r^{(l)}) \\ &= -(\mathbf{1_r^{(k)}} - \mathbf{1_r^{(l)}})\mathbf{x_{ur}} \end{aligned}$$

(K-1) double differences (here, satellite 1 is reference):

$$\begin{bmatrix} \phi_{ur}^{(21)} \\ \phi_{ur}^{(31)} \\ \vdots \\ \phi_{ur}^{(K1)} \end{bmatrix} = \lambda^{-1} \begin{bmatrix} -(\mathbf{1_r^{(1)}} - \mathbf{1_r^{(1)}})^T \\ -(\mathbf{1_r^{(2)}} - \mathbf{1_r^{(1)}})^T \\ \vdots \\ -(\mathbf{1_r^{(K)}} - \mathbf{1_r^{(1)}})^T \end{bmatrix} \mathbf{x_{ur}} + \begin{bmatrix} N_{ur}^{(21)} \\ N_{ur}^{(31)} \\ \vdots \\ N_{ur}^{(K1)} \end{bmatrix} + \begin{bmatrix} \epsilon_{\phi,ur}^{(21)} \\ \epsilon_{\phi,ur}^{(31)} \\ \vdots \\ \epsilon_{\phi,ur}^{(K1)} \end{bmatrix}$$

where $b_{ur} = c\delta t_{ur}$ is receiver clock bias

Triple Difference

Triple Difference

- adds difference in time
- difference double difference from epoch t_1 and t_0
- can be used to eliminate phase ambiguity
- but removes most of geometric strength and hence gives weak positions