ERTH 491-01 / GEOP 572-02
Geodetic Methods

– Lecture 08: GPS Ambiguity Resolution, Error Sources/Models –

Ronni Grapenthin
rg@nmt.edu
MSEC 356
x5924

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Carrier Phase Measurement

$$\phi = \frac{1}{\lambda} * (r + I + T) + f * (\delta t_u - \delta t^s) + N + \epsilon_\phi$$

(units of cycles) where

- λ, f - carrier wavelength, frequency
- r - geometric range
- I, T - ionospheric, tropospheric propagation errors (path delays)
- $\delta t_u, \delta t^s$ - receiver, satellite clock biases
- N - phase ambiguity
- ϵ_ϕ - error term (phase)

compare to code measurement eqn (units of distance):

$$\rho = r + I + T + c * (\delta t_u - \delta t^s) + \epsilon_\rho$$

Code tracking: unambiguous (long!)
$\sigma(\epsilon_\rho) \approx 0.5\,\mathrm{m}$
$\sigma(\epsilon_\phi) \approx 0.025\,\mathrm{cycle}\ (5\,\mathrm{mm})$

Integer Ambiguity Resolution 1/5

- uncertainty in integer estimation depends on carrier wavelength
- increase wavelength –> decrease uncertainty: create *wide lane measurement*:

$$\begin{aligned}
\phi_{L12} &= \phi_{L1} - \phi_{L2} \\
&= r(f_{L1} - f_{L2})/c + (N_{L1} - N_{L2}) + \epsilon_{\phi_{L12}} \\
&= r/\lambda_{L12} + N_{L12} + \epsilon_{\phi_{L12}}
\end{aligned}$$

where $\lambda_{L12} = c/(f_{L1} - f_{L2}) = 0.862m$, N_{L12} is integer ambiguity

Integer Ambiguity Resolution 2/5

Using

$$\rho_{L1} \quad = \quad r + \epsilon_{\rho_{L1}}$$

we can form estimate of N_{L12} as:

$$N_{L12} \quad \approx \quad \left[\phi_{L12} - \frac{\rho_{L1}}{\lambda_{L12}} \right]_{roundoff}$$

Here, $\sigma(N_{L12}) \approx 1.2$ cycles;
compared to $\sigma(N_{L1}) \approx 5$ cycles

Integer Ambiguity Resolution 3/5

- wide lane measurements much noisier than L1,L2 measurements
- *narrow lane combination* $\phi_{Ln} = \phi_{L1} + \phi_{L2}$ less noisy
- though harder to resolve ambiguities with narrow lane
- position estimates would be more precise

Integer Ambiguity Resolution (one at a time) 4/5

With correct N_{L12} can determine N_{L1}, N_{L2}. Measurement eqs:

$$\begin{aligned} \phi_{L1} &= r/\lambda_{L1} + N_{L1} + \epsilon_{\phi_{L1}} \\ \phi_{L2} &= r/\lambda_{L2} + N_{L2} + \epsilon_{\phi_{L2}} \end{aligned}$$

we get

$$N_{L1} - \frac{\lambda_{L2}}{\lambda_{L1}} N_{L2} = \phi_{L1} - \frac{\lambda_{L2}}{\lambda_{L1}} \phi_{L2} + \epsilon$$

We have

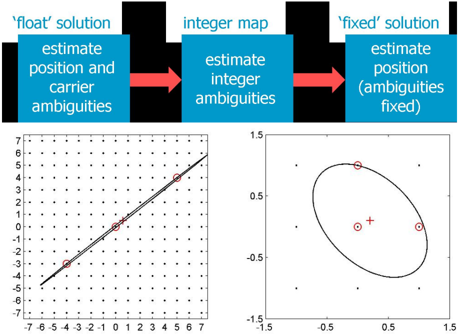
$$\begin{aligned} N_{L1} - N_{L2} &= N_{L12} \\ N_{L2} &= N_{L1} - N_{L12} \end{aligned}$$

So, we can solve for N_{L1}, N_{L2}:

$$N_{L1} = \left[\frac{\lambda_{L2}}{\lambda_{L1}} - 1 \right]^{-1} \left[\frac{\lambda_{L2}}{\lambda_{L1}} N_{L12} - \phi_{L1} + \frac{\lambda_{L2}}{\lambda_{L1}} \phi_{L2} \right]$$

Integer Ambiguity Resolution (as a set) 5/5

- 1) discard integer nature of ambiguities and find least squares 'float solution'
- 2) map to integer (decorrelate error elipse)
- 3) 'fixed solution': estimate position (other parameters) w/ integer ambiguities



http://www.citg.tudelft.nl/en/about-faculty/departments/geoscience-and-remote-sensing/research-themes/gps/lambda-method/

Ionospheric Delay 1/4

carrier phase (unit of cycles):

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Ionospheric Delay 2/4

- $\approx$ 50-1000 km above Earth
- ionized gases: free electrons and ions, sun's radiation/activity drives state
- composed of layers: D,E,F1,F2, peak electron density in F2 250-400 km height
- daily cycle with peak electron density at about 2 pm local time
- electron density 1-2 orders of magnitude difference between night/day
- changes w/ seasons, 11-year solar cycle, other short term anomalies (tsunamis), solar flares
- dispersive for GPS frequencies (different frequencies - different effective velocity)

Ionospheric Delay 3/4

Total electron content (TEC): number of electrons $n_e(l)$ in tube of 1 m^2 connecting satellite and receiver:

$$TEC = \int_R^S n_e(l)dl \text{ (TECU: TEC units)}$$

- *VTEC*: TEC in vertical direction, lowest TEC when satellite in zenith direction
- *VTEC* between 1-150 TECU
- region with highest ionospheric delay within $\pm 20^\circ$ of magnetic equator

Ionospheric Delay 4/4

Dual-frequency receivers allow (basically) elimination of ionosphere as source of error. Possible combinations for code range and carrier phases (derivation in Hofmann-Wellenhof et al.)

$$\begin{aligned} \rho_{L1,L2} &= \rho_{L1} - \frac{f_{L2}^2}{f_{L1}^2}\rho_{L2} \\ \phi_{L1,L2} &= \phi_{L1} - \frac{f_{L2}}{f_{L1}}\phi_{L2} \end{aligned}$$

other combinations possible.

Model exists to remove ionosphere from L1-only observations (*Klobuchar model*, [Klobuchar, 1996]).

Tropospheric Delay 1/3

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- refractivity of air parcel depends on temperature, partial pressures of dry gases, water vapor
- approximations: $N_d = 77.64P/T$ and $N_w = 3.73 \times 10^5 e/T^2$ with P total, e partial pressures (mB), T temperature in K.

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- lots of both exist

Receiver Noise

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Receiver Noise

- RF radiation sensed by antenna (interference)
- noise introduced by antenna, amplifiers, cables (!), receiver, signal quantization!
- absence of interference: rcx sees waveform = GPS + random noise
- fine structure of signal can be masked by noise (esp. low SNR)
- error varies w/ signal strength, which depends on satellite elev angle

Multipath

carrier phase (unit of cycles):

$$\phi = \frac{1}{\lambda} * (r + I + T) + f * (\delta t_u - \delta t^s) + N + \mathbf{MP} + \epsilon_\phi$$

code measurement eqn (units of distance):

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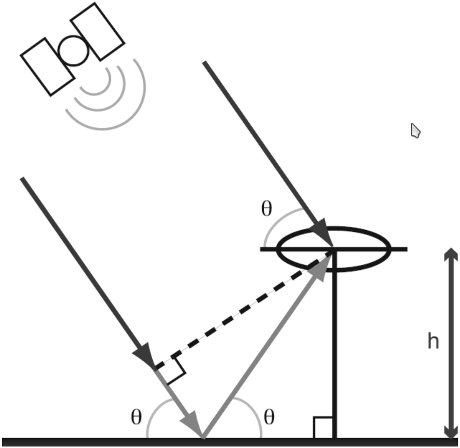
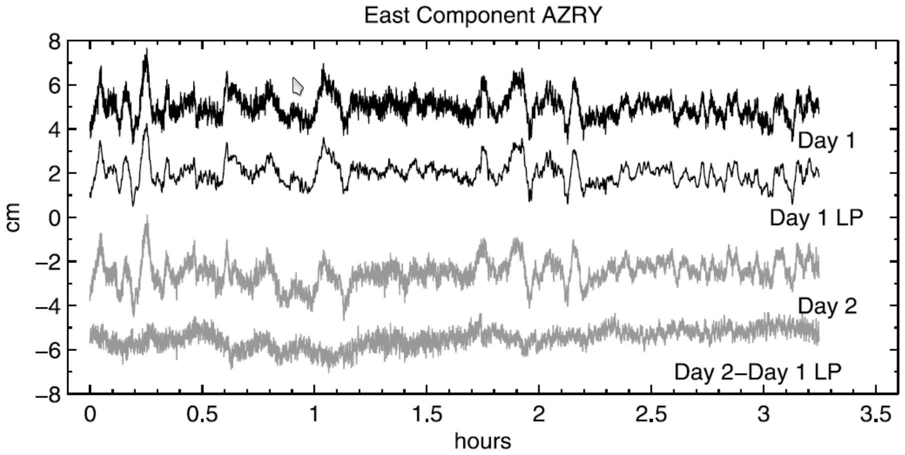
Multipath



Larson et al. (2007)

- best seen in subdaily solutions
- signal reaches antenna via direct and indirect paths
- reflected signal delayed, weaker
- mitigation: antenna design, receiver algorithms
- code and phase measurement are sum of received signals
- pseudorange: 1-5 m error
- phase: 1-5 cm error (no worse than 1/4 cycle)

Eliminating Multi-Path through Sidereal Filtering



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