ERTH 491-01 / GEOP 572-02
Geodetic Methods

– Lecture 12: InSAR - Making the Interferogram –

Ronni Grapenthin
rg@nmt.edu
MSEC 356
x5924

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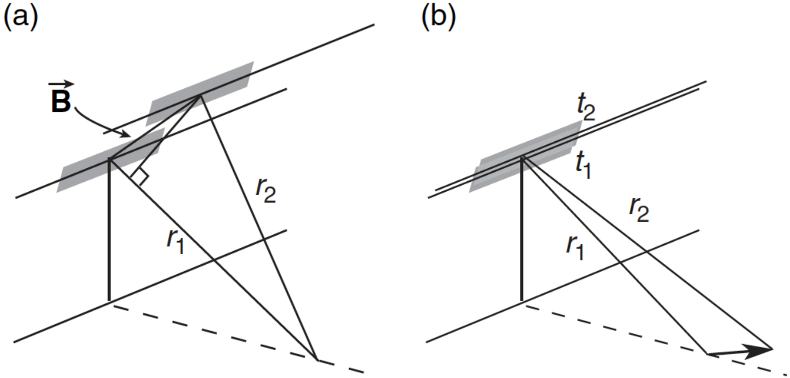
Difference between InSARs

Topo: look at same thing from 2 views (SRTM)

Deformation: look at same thing from same point and see whether it moved



Simons and Rosen, 2007

Making an Interferogram

Make interferogram from 2 Single Look Complex images
(images are in radar coordinates: range ρ, azimuth a):

1. align reference and repeat images to sub-pixel accuracy
2. multiply complex images (SLC) to form complex interferogram
3. extract phase: $\phi_2 - \phi_1 = arctan\frac{Im}{Re}$

Making an Interferogram: Step 1 - Alignment

- take 100s of small sub-patches (e.g. 64×64) from master and slave
- 2D cross correlation of patch pairs
- determine 6-parameter affine transformation to align slave to master image
- affine: parallel remains, straight remains, points preserved

Making an Interferogram: Step 2 - Multiply

Complex number of each pixel, $C(x)$, in terms of amplitude, $A(x)$, and phase, $\phi(x)$:

$$C(x) = A(x)e^{i\,\phi(x)}$$

with $x = (\rho, a)$

Making an Interferogram: Step 2 - Multiply

Complex number of each pixel, $C(x)$, in terms of amplitude, $A(x)$, and phase, $\phi(x)$:

$$C(x) = A(x)e^{i\,\phi(x)}$$

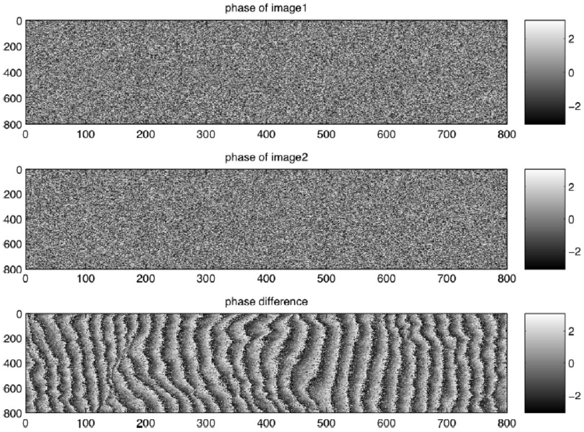
with $x = (\rho, a)$

Multiply (pixel by pixel, note complex conjugate!):

$$\begin{aligned} C_2\,C_1^* &= A_2\,A_1\,e^{i(\phi_2-\phi_1)} \\ &= Re(x) + i\,Im(x) \end{aligned}$$

Making an Interferogram: Step 3 - Get Phase

$$\phi_2 - \phi_1 = \mathit{arctan}\frac{Im}{Re}$$



Sandwell et al., 2011, GMTSAR documentation

Phase Contributors

What's in the phase?

Phase Contributors

$$\phi = E + \phi_{topo} + D + \epsilon_{orbit} + I + T + \epsilon$$

where:

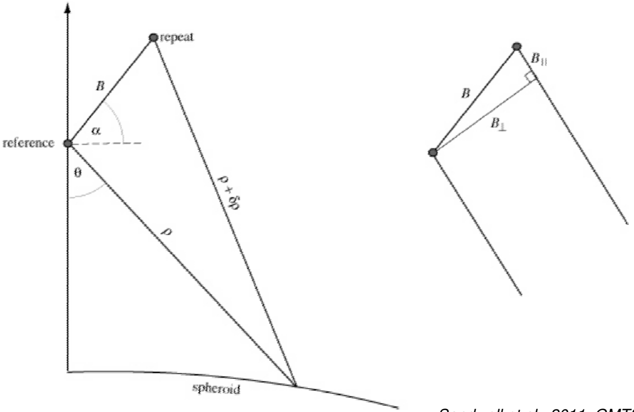
- E: earth curvature (almost planar, known)
- ϕ_{topo}: topographic phase (broad spectrum)
- **D: surface deformation (unknown, we want to know!)**
- ϵ_{orbit}: orbit error (almost a plane, mostly known)
- I: Ionospheric Delay (plane or 4 km wavelength waves!)
- T: Tropospheric Delay (power law, unknown)
- ϵ: phase noise (white, unknown)

Correct for Earth's Shape

EarthShape = curvature + topography

Phase due to Earth Curvature

Repeat-pass interferometry geometry:



Sandwell et al., 2011, GMTSAR documentation

assume parallel paths:

$$
\begin{aligned}
B_{\|} &= B\,sin(\theta - \alpha) \\
B_{\perp} &= B\,cos(\theta - \alpha)
\end{aligned}
$$

Phase due to Earth Curvature

Issue?

Phase due to Earth Curvature

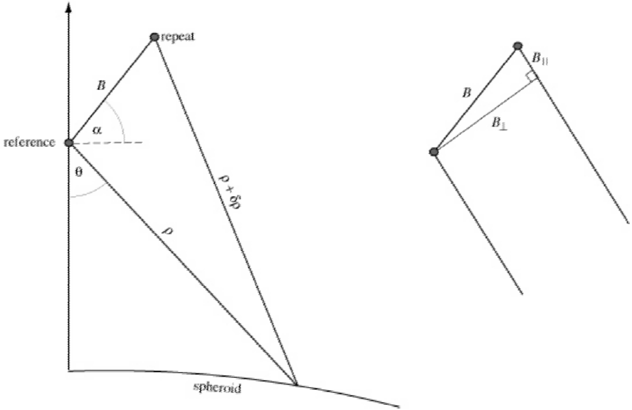
Issue?

- θ (look angle) changes across path
- single $B_{\perp}$ for combination is approximate!

Phase due to Earth Curvature

Phase difference ϕ relates to range difference $\delta\rho$:

$$\phi = \frac{4\,\pi}{\lambda}\delta\rho$$

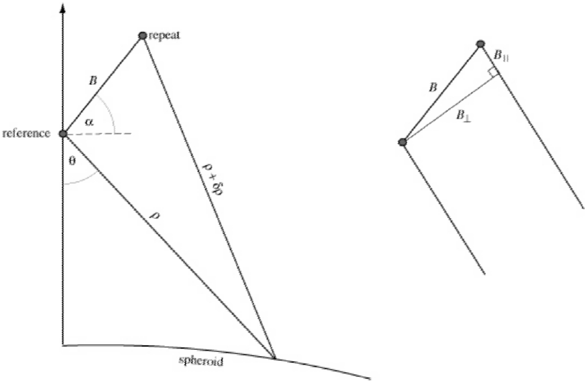


Sandwell et al., 2011, GMTSAR documentation

Phase due to Earth Curvature

Law of Cosines:

$$(\rho + \delta\rho)^2 \quad = \quad B^2 + \rho^2 - 2\,B\,\rho\,sin(\theta - \alpha)$$



Sandwell et al., 2011, GMTSAR documentation

Phase due to Earth Curvature

Law of Cosines:

$$(\rho + \delta\rho)^2 = B^2 + \rho^2 - 2\,B\,\rho\,sin(\theta - \alpha)$$

Algebra:

$$\begin{aligned}
(\rho + \delta\rho)^2 &= B^2 + \rho^2 - 2\,B\,\rho\,sin(\theta - \alpha) \\
2\rho\delta\rho + \delta\rho^2 &= B^2 - 2\,B\,\rho\,sin(\theta - \alpha) \\
2\rho(\delta\rho + \frac{\delta\rho^2}{2\rho}) &= 2\rho(\frac{B^2}{2\rho} - B\,sin(\theta - \alpha)) \\
\delta\rho &= -B\,sin(\theta - \alpha) + \frac{1}{2\rho}(B^2 + \delta\rho^2)
\end{aligned}$$

Phase due to Earth Curvature

Law of Cosines (after algebra):

$$\delta\rho \quad = \quad -B\,sin(\theta - \alpha) + \frac{1}{2\rho}(B^2 + \delta\rho^2)$$

Assume $\delta\rho << \rho$:

$$\delta\rho \quad = \quad \frac{B}{2\,\rho} - B\,sin(\theta - \alpha)$$

Assume $B << \rho$:

$$\delta\rho \quad = \quad -B\,sin(\theta - \alpha)$$

Plug into phase difference eqn:

$$\phi \quad = \quad \frac{-4\,pi}{\lambda} B\,sin(\theta - \alpha)$$

Phase due to Earth Curvature

$$\phi = \frac{-4\,\mathit{pi}}{\lambda} B\,\mathit{sin}(\theta - \alpha)$$

- Making some more assumptions we could get phase correction based on $\delta\rho << \rho$ assumption
- instead, use phase expression to approximate higher order terms
- need ot think about phase change wrt to range first

Phase due to Earth Curvature

Differentiate w.r.t. range:

$$\frac{\delta\phi}{\delta\rho} = \frac{-4\,pi}{\lambda} B\,cos(\theta-\alpha)\frac{\delta\theta}{\delta\rho}$$
$$= \frac{-4\,pi}{\lambda} B_{\perp}\frac{\delta\theta}{\delta\rho}$$

Depends on $B_{\perp}$ and derivative of look angle wrt range: $\frac{\delta\theta}{\delta\rho}$

Look angle wrt range

Need $\frac{\delta\theta}{\delta\rho}$:

- assume locally spherical Earth
- adjust local radius, $r_e(\psi)$, of the Earth using WGS-84 ellipsoid
- large differences between spherical and elliptical Earth for long SAR swaths!
- would give fringes (artifacts) that depend on baseline
- get correction term later!

$$r_e(\psi) = \left(\frac{cos^2(\psi)}{a^2} + \frac{sin^2(\psi)}{c^2}\right)^{-\frac{1}{2}}$$

- ψ: latitude
- a: equatorial radius 6378 km
- c: polar radius 6357 km

Exact Topography

$$\delta\rho = -B\,sin(\theta - \alpha) + \frac{1}{2\rho}(B^2 + \delta\rho^2)$$

…keep higher order term, but use $\delta\rho = -B\,sin(\theta - \alpha)$ found above

$$\begin{aligned}\delta\rho &= -B\,sin(\theta - \alpha) + \frac{B^2}{2\rho}(1 - sin^2(\theta - \alpha)) \\ &= -B\,sin(\theta - \alpha) + \frac{B^2}{2\rho}(cos^2(\theta - \alpha))\end{aligned}$$

…could iterate to get even more accurate …

Exact Topography

ratio of first and second order terms:

$$\textit{ratio} = \frac{B\,\textit{cos}^2(\theta - \alpha)}{2\rho\,\textit{sin}(\theta - \alpha)}$$
$$= \frac{3\,B}{4\,\rho}$$

…if we assume $\alpha = 0$, $\theta = 30^\circ$

Topography Correction Algorithm

- map topography from lat, lon, height to radar coordinates and topography over range, azimuth $t(\rho, a)$

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- interpolate topography to each range pixel get look angle from:

$$\theta_{\rho,a} = cos^{-1}\left[\frac{(b^2 + rho^2 - (r_e + t(\rho, a)))^2}{2\rho\, b}\right]$$

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- with look angle for each range pixel, calculate phase correction for repeat image:

$$\phi_{\rho,a} \quad = \quad -\frac{4\,\pi\,B}{\lambda} sin(\theta_{\rho,a}) - \alpha) + \frac{2\,\pi\,B^2}{\lambda\rho} cos^2(\theta_{\rho,a} - \alpha)$$

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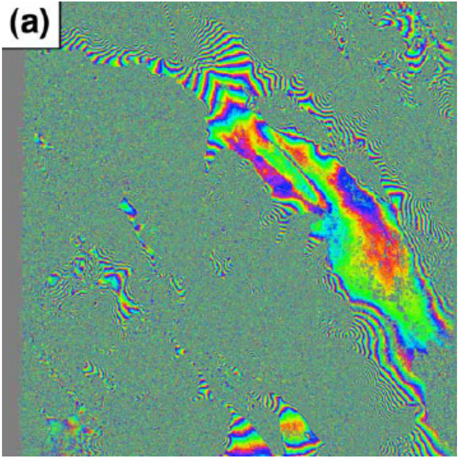
- multiply $C_2\, C_1^*$
- extract phase difference $\phi_2 - \phi_1 = arctan(\frac{Im}{Re})$

Phase due to Topography

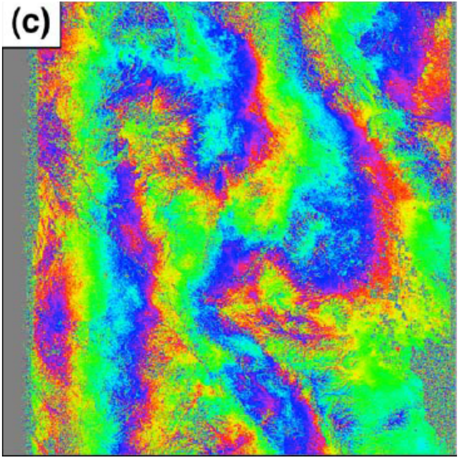
1.95 km baseline Interferogram,
no topo removed (120 fringes need removal):



Sandwell et al., 2011, GMTSAR documentation

Phase due to Topography

1.95 km baseline Interferogram,
topography correction using **approximate** formulas:



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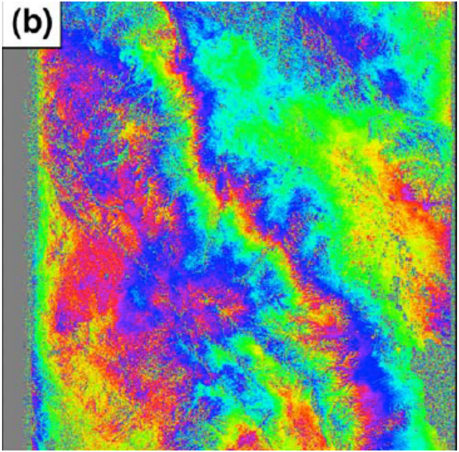
Phase due to Topography

1.95 km baseline Interferogram,
topography correction using **exact(er)** formulas:



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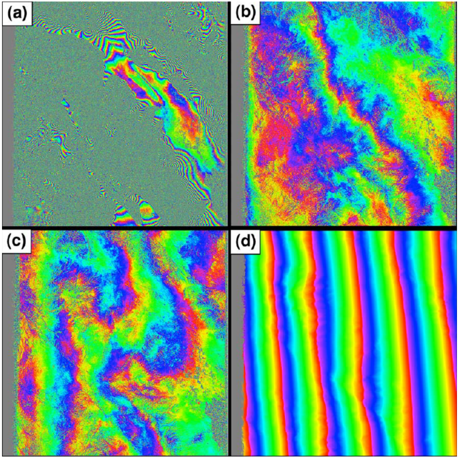
Phase due to Topography

1.95 km baseline Interferogram,

Difference between exact and approx. formulas = 0.6 m ramp



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