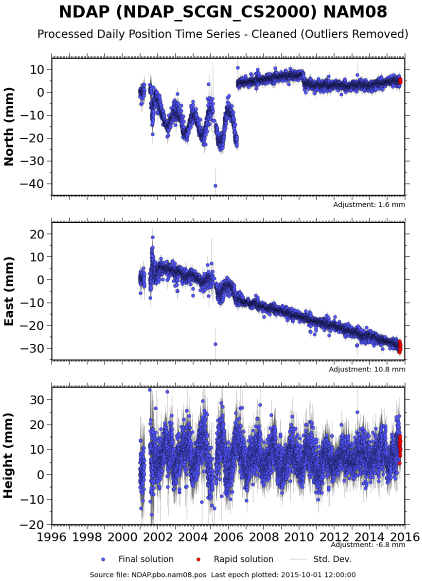
ERTH 491-01 / GEOP 572-02
Geodetic Methods

– Lecture 13: InSAR - Unwrapping the Phase –

Ronni Grapenthin
rg@nmt.edu
MSEC 356
x5924

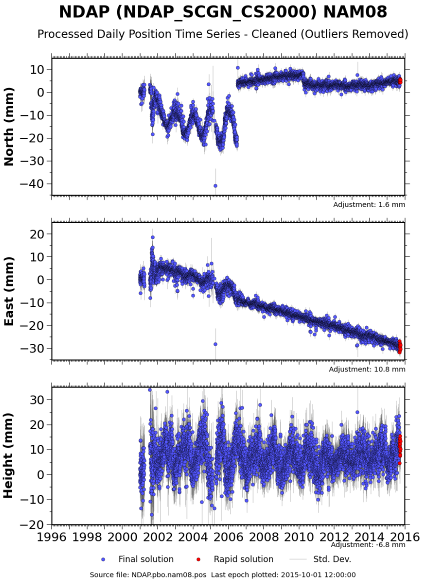
October 05, 2015

New Segment: “Guess the Process”



source: *UNAVCO*

New Segment: “Guess the Process”



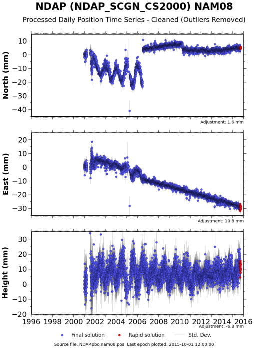
NDAP (NDAP_SCGN_CS2000) NAM08

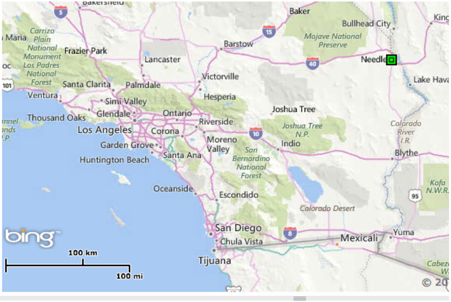
Processed Daily Position Time Series - Cleaned (Outliers Removed)

Source file: NDAP.pbo.nam08.pos Last epoch plotted: 2015-10-01 12:00:00

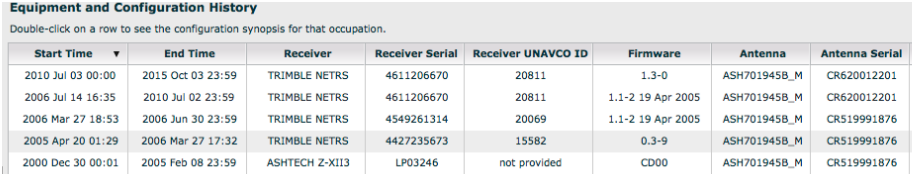
source: *UNAVCO*

New Segment: “Guess the Process”





source: *UNAVCO*



Equipment and Configuration History

Double-click on a row to see the configuration synopsis for that occupation.

Start Time ▼	End Time	Receiver	Receiver Serial	Receiver UNAVCO ID	Firmware	Antenna	Antenna Serial
2010 Jul 03 00:00	2015 Oct 03 23:59	TRIMBLE NETRS	4611206670	20811	1.3-0	ASH701945B_M	CR620012201
2006 Jul 14 16:35	2010 Jul 02 23:59	TRIMBLE NETRS	4611206670	20811	1.1-2 19 Apr 2005	ASH701945B_M	CR620012201
2006 Mar 27 18:53	2006 Jun 30 23:59	TRIMBLE NETRS	4549261314	20069	1.1-2 19 Apr 2005	ASH701945B_M	CR519991876
2005 Apr 20 01:29	2006 Mar 27 17:32	TRIMBLE NETRS	4427235673	15582	0.3-9	ASH701945B_M	CR519991876
2000 Dec 30 00:01	2005 Feb 08 23:59	ASHTECH Z-XII3	LP03246	not provided	CD00	ASH701945B_M	CR519991876

InSAR - Processing Flow

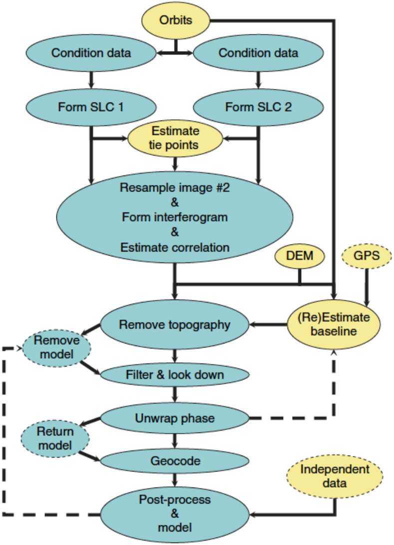
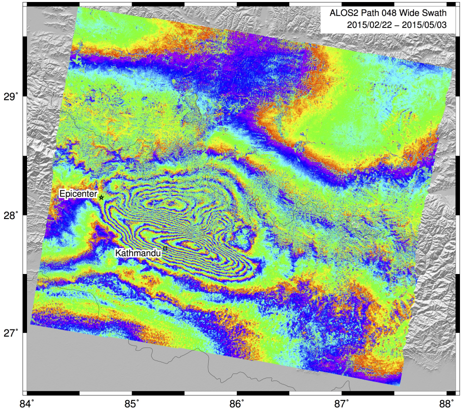


Figure 6 Representative differential InSAR processing flow diagram. Blue bubbles represent image output, yellow ellipses represent nonimage data. Flow is generally down the solid paths, with optional dashed paths indicating potential iteration steps. DEM, digital elevation model; SLC, single look complex image.

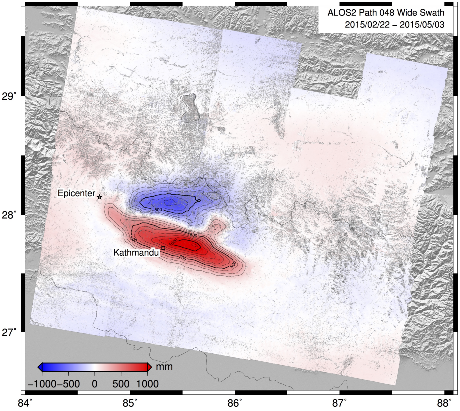
Simons and Rosen, 2007

InSAR - Phase Unwrapping

Getting from here . . .



. . . to here



Lindsey et al., GRL, 2015

InSAR - Phase Unwrapping

Materials for this lecture come mostly from:

- Goldstein, R., Zebker, H., and Werner, C. (1988). *Satellite radar interferometry- Two-dimensional phase unwrapping.* Radio science, 23(4), 713-720.
- Rosen, P., Hensley, S., Joughin, I. R., Li, F. K., Madsen, S. N., Rodriguez, E., and Goldstein, R. M. (2000). *Synthetic aperture radar interferometry.* Proceedings of the IEEE, 88(3), 333-382.
- Chen, C. W. and Zebker, H. A. (2001). *Two-dimensional phase unwrapping with use of statistical models for cost functions in nonlinear optimization.* JOSA A, 18(2), 338-351.
- Hooper, A. and Zebker, H. A. (2007). *Phase unwrapping in three dimensions with application to InSAR time series.* JOSA A, 24(9), 2737-2747.

InSAR - Phase Unwrapping

- remove modulo-2π ambiguity
- classes of algorithms:
    - integration with branch cuts
    - *L*-norm minimization (fit unwrapped solution to gradients of wrapped phase, minimize cost function)
    - mixed L-norms + probabilistic approach (snaphu)
    - 2D, 3D (where third dimension is time)

InSAR - Phase Unwrapping: Naive Approach

- assume neighboring phase values vary slowly: within one half-cycle (π rad)
- integrate phase differences from point to point
- add integer number of cycles that minimized phase differences
- 1D example (unit: cycles): 0.5, 0.6, 0.7, 0.8, 0.9, 0.0, 0.1, 0.2 ...
- clearly need to add 1 cycle to last 3 values

What are possible unwrapping errors?

InSAR - Phase Unwrapping: Naive Approach

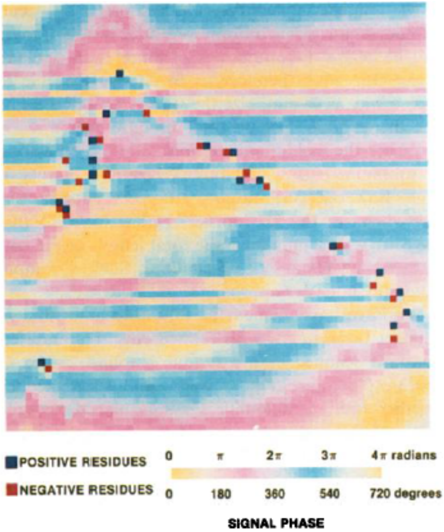
- assume neighboring phase values vary slowly: within one half-cycle (π rad)
- integrate phase differences from point to point
- add integer number of cycles that minimized phase differences
- 1D example (unit: cycles): 0.5, 0.6, 0.7, 0.8, 0.9, 0.0, 0.1, 0.2 $\ldots$
- clearly need to add 1 cycle to last 3 values

What are possible unwrapping errors?

- local errors: a few points are noise-corrupted
- global errors: local error propagates through sequence

Problem: Errors or phase variations $> \pi$ make integration path dependent!

InSAR - Phase Unwrapping: Naive Approach



Goldstein et al., JGR, 1988

InSAR - Phase Unwrapping: Branch Cut

- evaluate clock-wise sum of adjacent points:

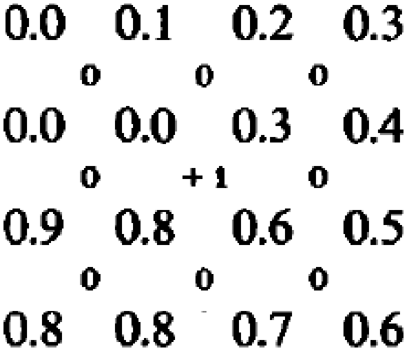
$$\begin{tikzcd}
0.0 \arrow[r] & 0.3 \arrow[d] \\
0.8 \arrow[u] & 0.6 \arrow[l]
\end{tikzcd}$$

Goldstein et al., JGR, 1988

- zero ± 1 cycle if phase difference consistent with half-cycle assumption
- inconsistencies with half-cycle assumption indicated by non-zero results
- such “residues” are either positively or negatively “charged” (depending on sign of sum)

InSAR - Phase Unwrapping: Branch Cut

- integration paths that enclose **single residue** have **inconsistency** in unwrapped phase
- integration paths that enclose equal number of plus and minus residue have **no inconsistency**
- when residues identified: consistent unwrapping possible



0.0		0.1		0.2		0.3
	0		0		0	
0.0		0.0		0.3		0.4
	0		+1		0	
0.9		0.8		0.6		0.5
	0		0		0	
0.8		0.8		0.7		0.6

Goldstein et al., JGR, 1988

InSAR - Phase Unwrapping: Branch Cut

- “branch cuts” between residues prevent integration path from crossing
- various (fully automated) strategies to choose cuts (e.g., minimize total discontinuity)

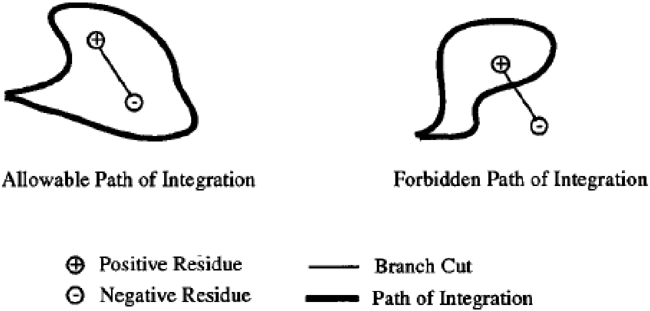
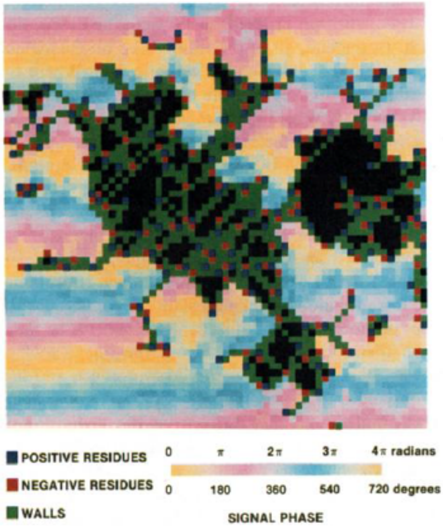


Fig. 18. An example of a branch cut and allowable and forbidden paths of integration.

Rosen et al., Proc. IEEE, 2000

InSAR - Phase Unwrapping: Branch Cut



Goldstein et al., JGR, 1988

Cuts in place, not yet integrated

InSAR - Phase Unwrapping: Branch Cut

Problem: How to select cuts?

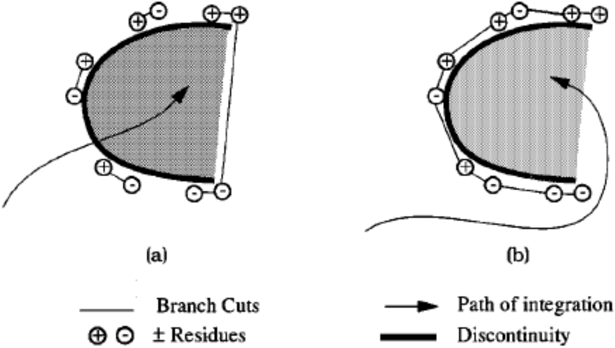


Fig. 19. Cut dependencies of unwrapped phase: (a) shortest path cuts and (b) better choice of cuts.

Rosen et al., Proc. IEEE, 2000

InSAR - Phase Unwrapping: L-norm minimization

Minimize (2D-range-azimuth coordinate system):

$$\sum_i \sum_j g_{ij}^{(r)}(\Delta\phi_{ij}^{(r)}, \Delta\psi_{ij}^{(r)}) + \sum_i \sum_j g_{ij}^{(a)}(\Delta\phi_{ij}^{(a)}, \Delta\psi_{ij}^{(a)})$$

InSAR - Phase Unwrapping: L-norm minimization

Minimize (2D-range-azimuth coordinate system):

$$\sum_i \sum_j g_{ij}^{(r)}(\Delta\phi_{ij}^{(r)}, \Delta\psi_{ij}^{(r)}) + \sum_i \sum_j g_{ij}^{(a)}(\Delta\phi_{ij}^{(a)}, \Delta\psi_{ij}^{(a)})$$

- $\Delta\phi^{(r)}$, $\Delta\psi^{(r)}$: **range** component of wrapped, unwrapped (and rewrapped) phase gradients
- $\Delta\phi^{(a)}$, $\Delta\psi^{(a)}$: **azimuth** component of wrapped, unwrapped phase gradients
- e.g, $\Delta\phi_{ij}^{(r)} = \phi_{i,j} - \phi_{i-1,j}$, analog for azimuth, unwrapped components

InSAR - Phase Unwrapping: L-norm minimization

Minimize (2D-range-azimuth coordinate system):

$$\sum_i \sum_j g_{ij}^{(r)}(\Delta\phi_{ij}^{(r)}, \Delta\psi_{ij}^{(r)}) + \sum_i \sum_j g_{ij}^{(a)}(\Delta\phi_{ij}^{(a)}, \Delta\psi_{ij}^{(a)})$$

- $\Delta\phi^{(r)}$, $\Delta\psi^{(r)}$: **range** component of wrapped, unwrapped (and rewrapped) phase gradients
- $\Delta\phi^{(a)}$, $\Delta\psi^{(a)}$: **azimuth** component of wrapped, unwrapped phase gradients
- e.g, $\Delta\phi_{ij}^{(r)} = \phi_{i,j} - \phi_{i-1,j}$, analog for azimuth, unwrapped components

Cost-function often restricted in form:

$$g_{ij}(\Delta\phi, \Delta\psi) = w_{ij}|\Delta\phi_{ij} - \Delta\psi_{ij}|^P$$

- all cost functions have same shape determined by constant P ($P = 2$: Least squares problem)
- indep. weights w determine each cost function's contribution

InSAR - Phase Unwrapping: L-norm minimization

- all cost functions have same shape determined by constant P ($P = 2$: Least squares problem)
- indep. weights w determine each cost function's contribution

InSAR - Phase Unwrapping: L-norm & Probabilistic

- no physical reasons that optimal L^P solution must be correct
- *Chen & Zebker, JOSA, 2001* introduce objective from generalized, statistical cost functions
- allow any form for cost function g
- allow g shape to vary for different parts of interferogram
- choose cost function that maximizes conditional probably of solution based on wrapped phase, image intensity, coherence
- **application-specific** cost functions
- solution **approximation** based on non-linear network optimization

InSAR - 3D Phase Unwrapping

- 3D: neighboring phase discontinuities form “disconitinuity surface” across which $\Delta\phi > \pi$

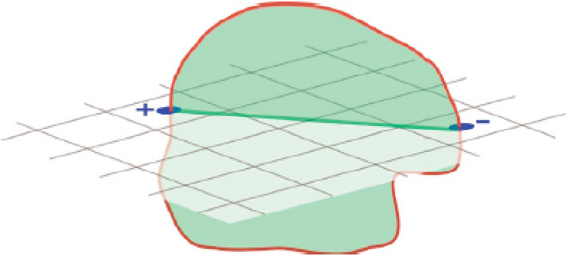
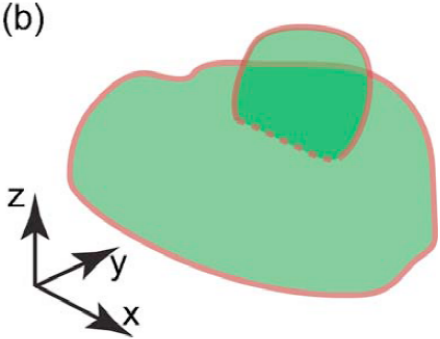


Fig. 1. (Color online) Simple phase discontinuity surface intersecting a 2D data set. The surface is bounded by a residue loop. Where the surface intersects the 2D data set results in a discontinuity line, which is bounded by a positive and a negative residue, arising where the residue loop intersects the data set.

Hooper & Zebker, JOSA, 2007

InSAR - 3D Phase Unwrapping

- branches in 3D:



Hooper & Zebker, JOSA, 2007

InSAR - 3D Phase Unwrapping

Minimize objective function of the form:

$$\sum_{ijk} g_{ijk}^{(r)}(\Delta\phi_{ijk}^{(r)}, \Delta\psi_{ijk}^{(r)}) + \sum_{ijk} g_{ijk}^{(a)}(\Delta\phi_{ijk}^{(a)}, \Delta\psi_{ijk}^{(a)}) + \sum_{ijk} g_{ijk}^{(t)}(\Delta\phi_{ijk}^{(t)}, \Delta\psi_{ijk}^{(t)})$$

- $\Delta\phi^{(t)}, \Delta\psi^{(t)}$ phase difference components in 3rd dimension
- summation in *t* direction over *k*
- $g_{ijk}(\Delta\phi_{ijk}, \Delta\psi_{ijk}) = w_{ijk}^{*}|\Delta\phi_{ijk}^{*} - \Delta\psi_{ijk}^{*}|^{P}$, focus on higher order P

InSAR - 3D Phase Unwrapping

Minimize objective function of the form:

$$\sum_{ijk} g_{ijk}^{(r)}(\Delta\phi_{ijk}^{(r)}, \Delta\psi_{ijk}^{(r)}) + \sum_{ijk} g_{ijk}^{(a)}(\Delta\phi_{ijk}^{(a)}, \Delta\psi_{ijk}^{(a)}) + \sum_{ijk} g_{ijk}^{(t)}(\Delta\phi_{ijk}^{(t)}, \Delta\psi_{ijk}^{(t)})$$

- $\Delta\phi^{(t)}, \Delta\psi^{(t)}$ phase difference components in 3rd dimension
- summation in *t* direction over *k*
- $g_{ijk}(\Delta\phi_{ijk}, \Delta\psi_{ijk}) = w_{ijk}^{*}|\Delta\phi_{ijk}^{*} - \Delta\psi_{ijk}^{*}|^{P}$, focus on higher order P

Hooper & Zebker (2007):

- L^{∞}-norm identical to L^{0} norm if only single cycle discontinuities exist
- General case (incl. multiple cycle discontinuities):
  - unwrap in 1D
  - iteratively improve on other 2 dimensions