Brief introduction to gravity methods: Some applications and a little theory

Lecture 1 of 2 for GEOP572

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What do we mean by gravity?

Measure variations in gravitational acceleration in space and/or time.

Gravitational acceleration (g) is not constant in space and time.

Newton’s Law from center of mass *m* to center of mass *M*:

$$\boldsymbol{F} = -G\frac{mM}{r^2}\hat{\boldsymbol{r}}$$

Substituting F = ma,

$$\boldsymbol{g} = \frac{\boldsymbol{F}}{m} = -G\frac{M}{r^2}\hat{\boldsymbol{r}}$$

Assuming continuous distribution of mass M in volume V with density ρ, then

$$\boldsymbol{g} = -G\int_V \frac{\rho(\boldsymbol{r}')}{r^2}\hat{\boldsymbol{r}}\,dV'$$

Thinking of Earth, where is the space and time dependence?

What do we mean by gravity?

Move the measurement mass, m, or the ‘mass-of-interest’, M, or change ρ through time.

$$\boldsymbol{g}(\boldsymbol{r},\boldsymbol{t}) = -G\int_V \frac{\rho(\boldsymbol{r}',\boldsymbol{t})}{r^2}\hat{\boldsymbol{r}}\,dV'$$

Thus, g can change at a given point through time by variations of density, or g can change in space by moving the measurement mass and emphasizing different parts of the volume V with the $1/r^2$ dependence.

How much does g **change in space and time?**

What do we mean by gravity?

How much does g **change in space and time? Well, that depends.**

g ≈ 9.8 m/s^2, or 980 Gal (for Galileo, fyi), or 980,000 mGal, or 980,000,000 µGal.

The Gal is the cgs (and most common) unit for gravitational acceleration in the gravity community.

Most variations are on the order of **0.01-10s of mGal.**

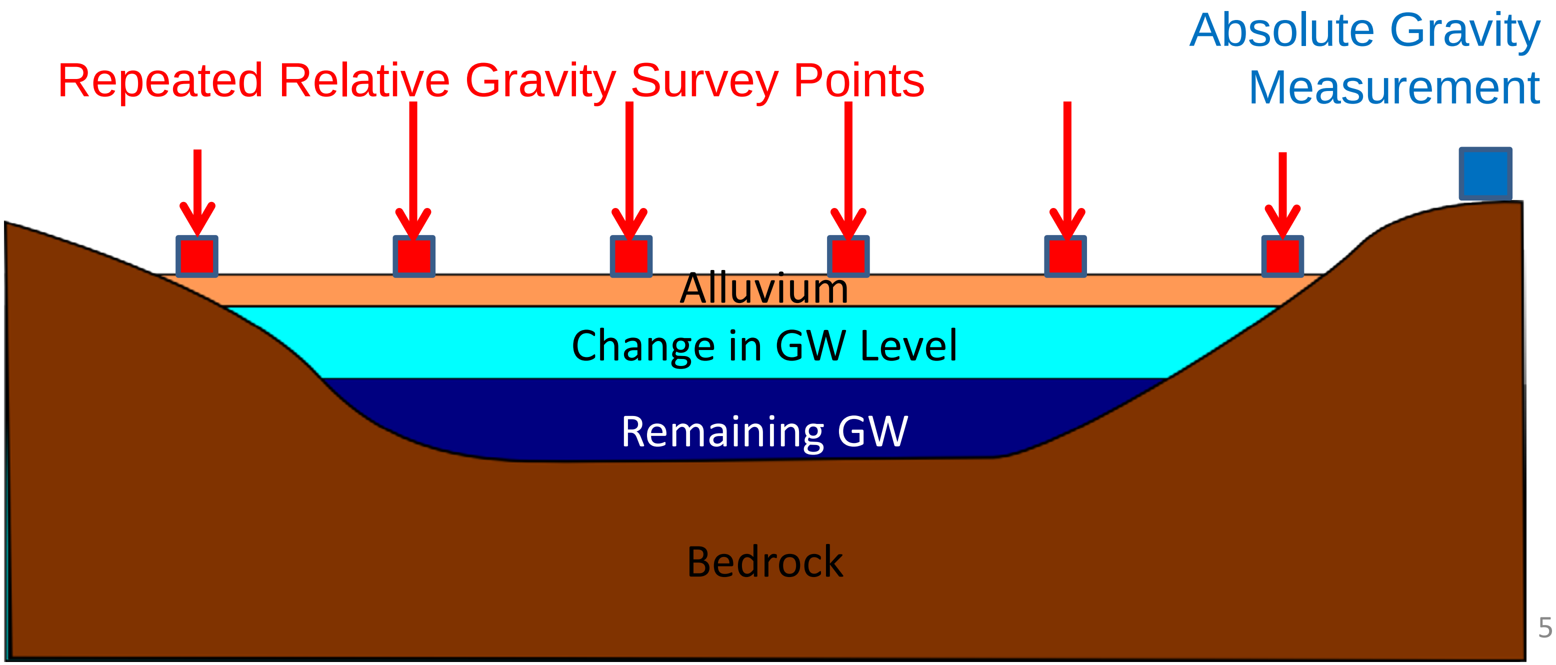
Local (100s of meters) spatial studies and terrestrial time-lapse surveys often require **less than 5 µGal resolution.** Thus, these surveys are commonly called “microgravity surveys”.

What is a ‘Gravity Measurement’?

- **A measurement of the change of the acceleration of gravity (relative), or**
- **A measurement of the acceleration of gravity (absolute).**

As water is removed, the acceleration of gravity underneath the relative gravimeter goes down with repeated measurements.

Needs to be tied to an absolute gravity reference station.



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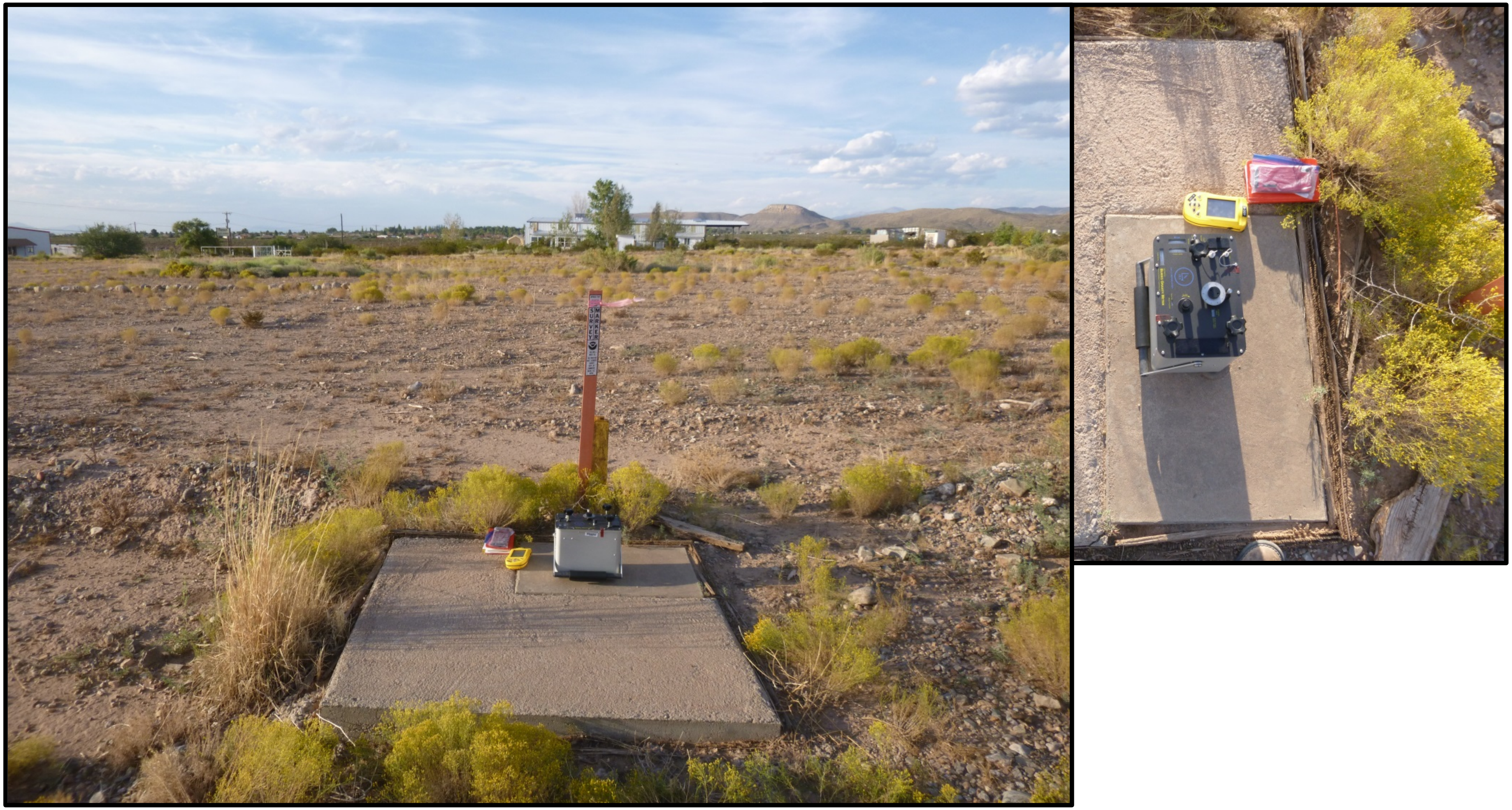


Some theoretical considerations

- **What does that integral combined with 1/r^2 dependence of *g* do to interpretation of spatial measurements?** *Answers: Superposition. Non-unique solutions. Able to determine the mass in sample volume.*

Gravitational potential:

$$\boldsymbol{g} = \boldsymbol{\nabla} U$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = \boldsymbol{\nabla}^{2} U$$

$$\boldsymbol{\nabla}^{2} U = -4\pi G\rho$$

$$U = G \int_{V} \frac{\rho}{r} dV$$

Some theoretical considerations

- **What does that integral combined with 1/r^2 dependence of *g* do to interpretation of spatial measurements?** *Answers: Superposition.*

$$\boldsymbol{g}(\boldsymbol{r},\boldsymbol{t}) = -G\int_V \frac{\rho(\boldsymbol{r}',\boldsymbol{t})}{r^2}\hat{\boldsymbol{r}}dV'$$

$$\boldsymbol{g}(\boldsymbol{r},\boldsymbol{t}) = -G\int_V \left(\frac{\rho_{bkgnd}\,(\boldsymbol{r}',\boldsymbol{t})}{r^2 1} + \frac{\rho_{anomaly}\,(\boldsymbol{r}',\boldsymbol{t})}{r^2}\right)\hat{\boldsymbol{r}}dV'$$

$$\boldsymbol{g}(\boldsymbol{r},t) - \boldsymbol{g}_{\boldsymbol{bkgnd}}(\boldsymbol{r},t) = \boldsymbol{g}_{\boldsymbol{anomaly}}(\boldsymbol{r},t)$$

If we can model or otherwise control for the background field (effects we are not interested in), then we can isolate ‘anomalous’ or target masses that deviate from background. Ore bodies, depth to bedrock, change in water storage, change in glacial mass, ….

Some theoretical considerations

- **What does that integral combined with $1/r^2$ dependence of *g* do to interpretation of spatial measurements?** *Answers: Able to determine the mass in sample volume..*

Very difficult way of estimating a deviatoric mass (e.g., ore body or water storage change or magma injection). After estimating the background mass distribution or gravitational field, define a large volume and use Gauss's law and an approximation that, for large distances, the gravitational potential is independent of the distribution of mass. With those assumptions, then

$$\int_{S_V} g_z dS = 2\pi G M_T$$

where S_v is the surface of the arbitrary background volume V, and M_T is the deviatoric mass. The g_z is the acceleration due to M_T. **Problem 1**: Assumes measurements are infinitely far away from M_T. **Problem 2**: Assumes that background field has been accurately accounted for.

With care, it is possible to estimate the mass of ore bodies or changes in water volume, or … somewhat independently of geometry.

Some theoretical considerations

- **What does that integral combined with $1/r^2$ dependence of *g* do to interpretation of spatial measurements?** *Answers: Non-unique solutions.*

- **Green's equivalent layer.** The gravitational field from any arbitrary bounded mass could also be caused by a thin layer of mass spread over any of its equipotential surfaces (i.e., surfaces of uniform **g**, or the shells coming out from the center of the earth).

The gravitational field from a mass does not uniquely map to the distribution of that mass. **Need outside information to constrain distribution.**

Implications of theory more clearly

- **Superposition of conservative field**
  - Final field is result of summing of many parts (many masses affect final measurement).
  - If non-desired components can be identified, then they can be subtracted. Then, the anomalous field from the mass of interest can be measured and interpreted.
- **Lack of uniqueness.**
  - Interpretation requires other knowledge to constrain gravity model.
  - Other knowledge (seismic, geologic judgement, etc.) has strong effect on gravity interpretation.
  - Assumptions of estimation of gravity anomaly vital to final interpretation.
- **Ability to estimate change in mass.**
  - If the background field can be removed, then the anomalous mass of a system can be measured, though with some difficulty.
  - Time-lapse measurements take the first field measurement as the base field and the subsequent measurements can show change in mass.

Quick, some examples of applications….

- Basin geometry from coarsely spaced gravity measurements.

- Estimates of recharge pattern from ephemeral river.

- Regional changes in gravity from satellites to estimate groundwater storage changes.

Albuquerque Basin Study of Grauch and Connell (2013)

- Survey stretches from just south of Santa Fe to just north of Socorro.
- With reasonably dense gravity surveys over 20+ years, required multiple geophysical surveys at low spatial resolution, plus deep bore holes to constrain subsurface geometry.

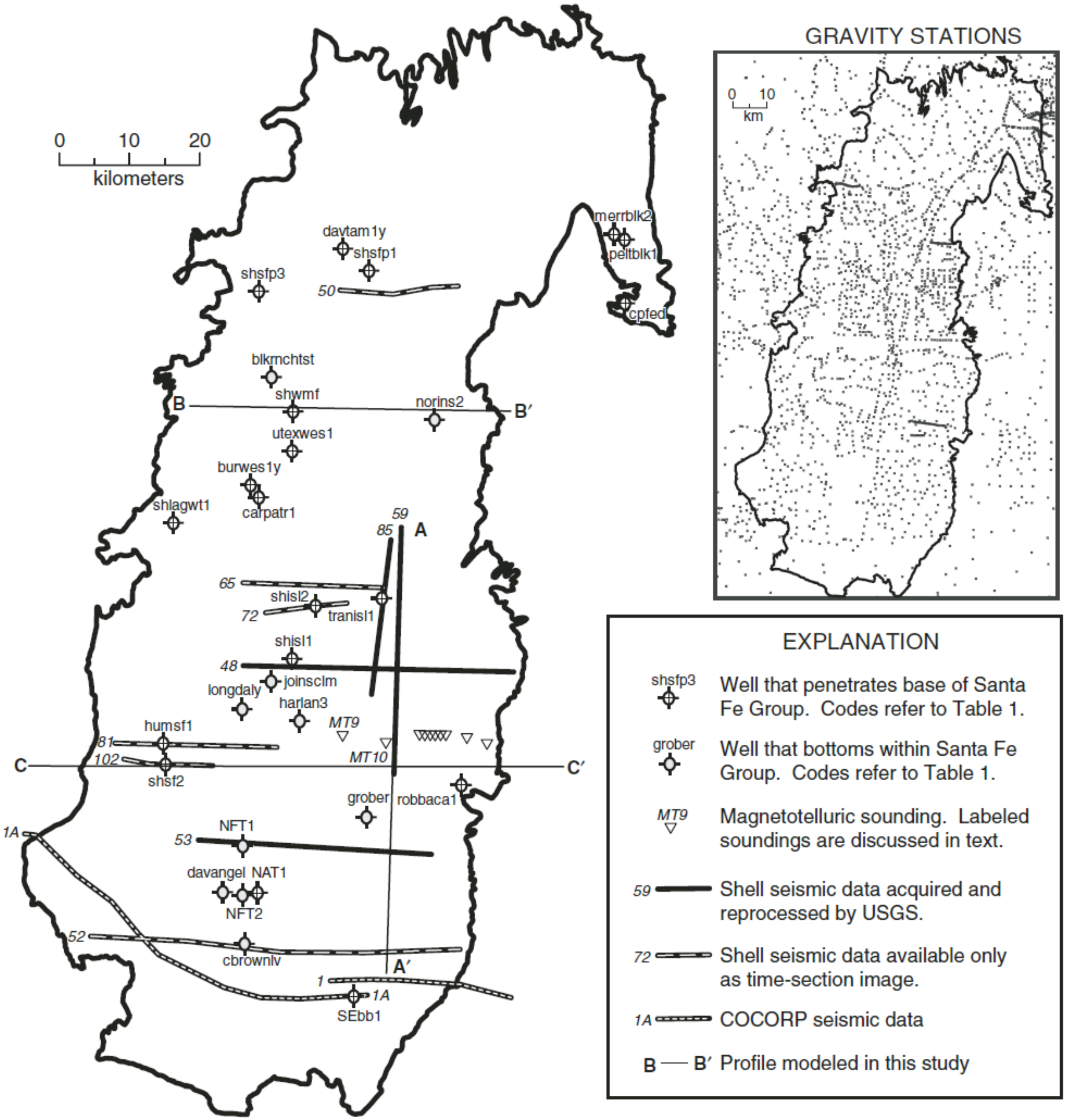


Figure 3. Locations and types of data sources. Basin boundary from Figure 1 is used for reference. Refer to Table 1 for well information and codes. Data sources for gravity, seismic-reflection, and magnetotelluric data are described in Appendices A and B (available on CD-ROM accompanying this volume and in the GSA Data Repository [see footnote 1]). High-resolution aeromagnetic data are presented separately in Appendix C. USGS—U.S. Geological Survey; COCORP—Consortium for Continental Reflection Profiling.

Gravity anomaly variations were on order of 100 mGal.

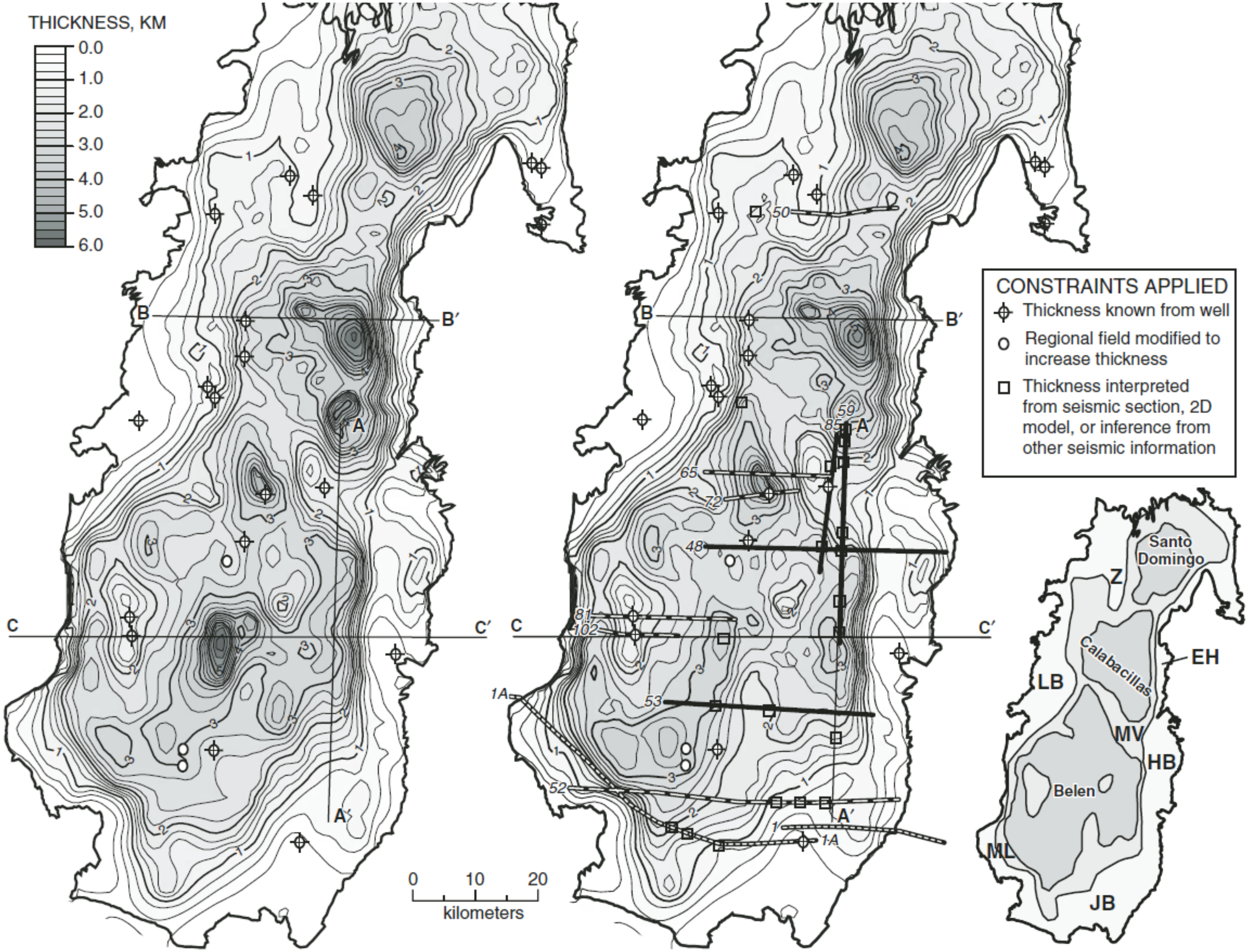
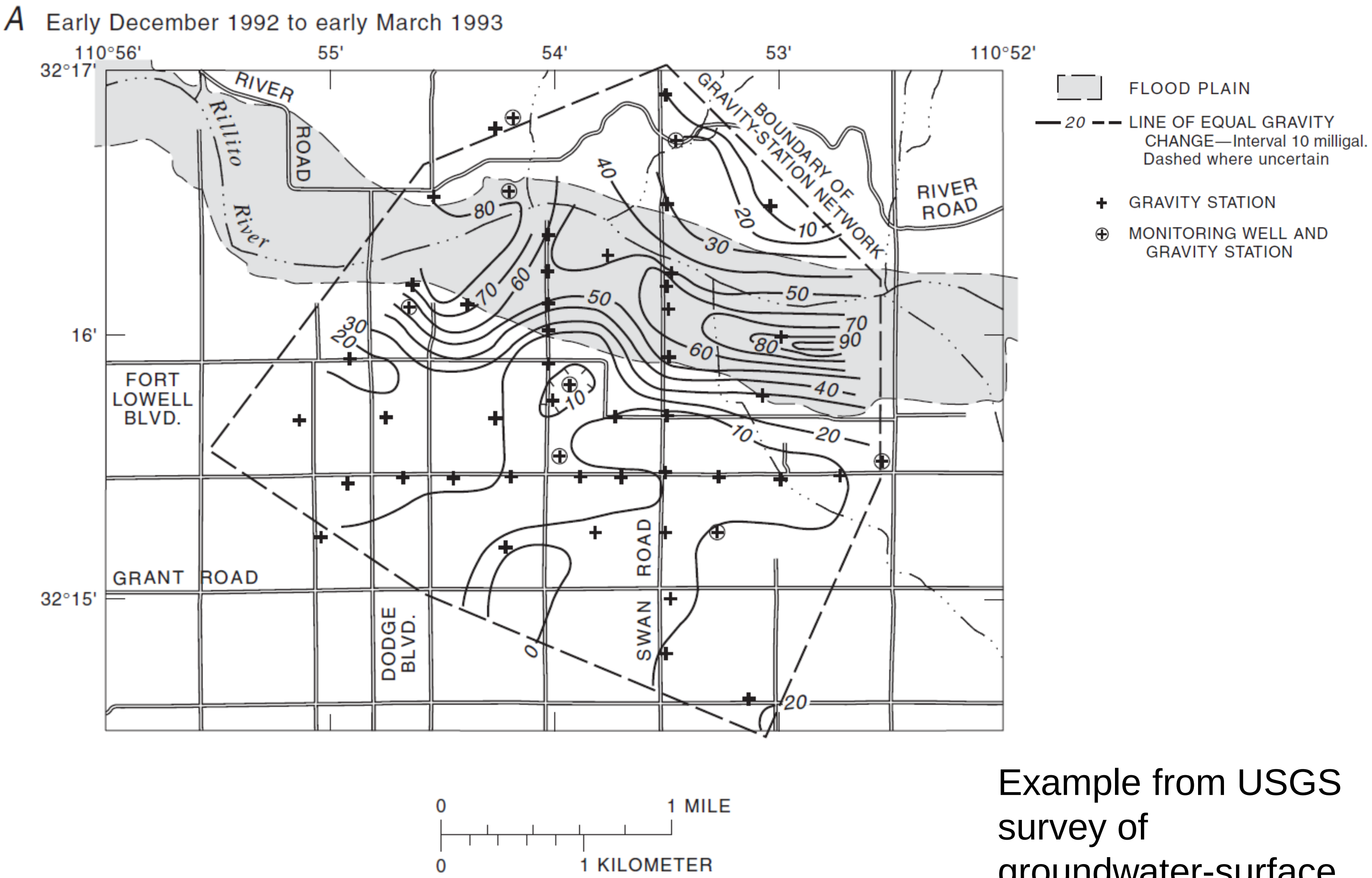


Figure 5. Isopach maps of the three-dimensional (3D) model of rift-fill thickness at initial and final stages of model development. Basin boundary of Figure 1 is shown for reference. The different types of constraint points are explained in text. (A) Isopachs from the initial 3D gravity model that shows what is required or permissible by the gravity data alone. (B) Isopachs from the final 3D geophysical model after all constraints are applied. Seismic lines (coded as in Fig. 3) are shown for reference. Inset shows shaded subbasin areas for reference on this and subsequent figures. The subbasins are generally outlined by the 1 km and 2 km isopach contours of the final 3D geophysical model (B). HB—Hubbell bench; JB—Joyita bench; LB—Laguna bench; ML—Monte Largo embayment; MV—Mountainview prong; EH—East Heights structural bench; Z—Ziana structure.

Examples of Gravity-based Storage Change Estimates



A Early December 1992 to early March 1993

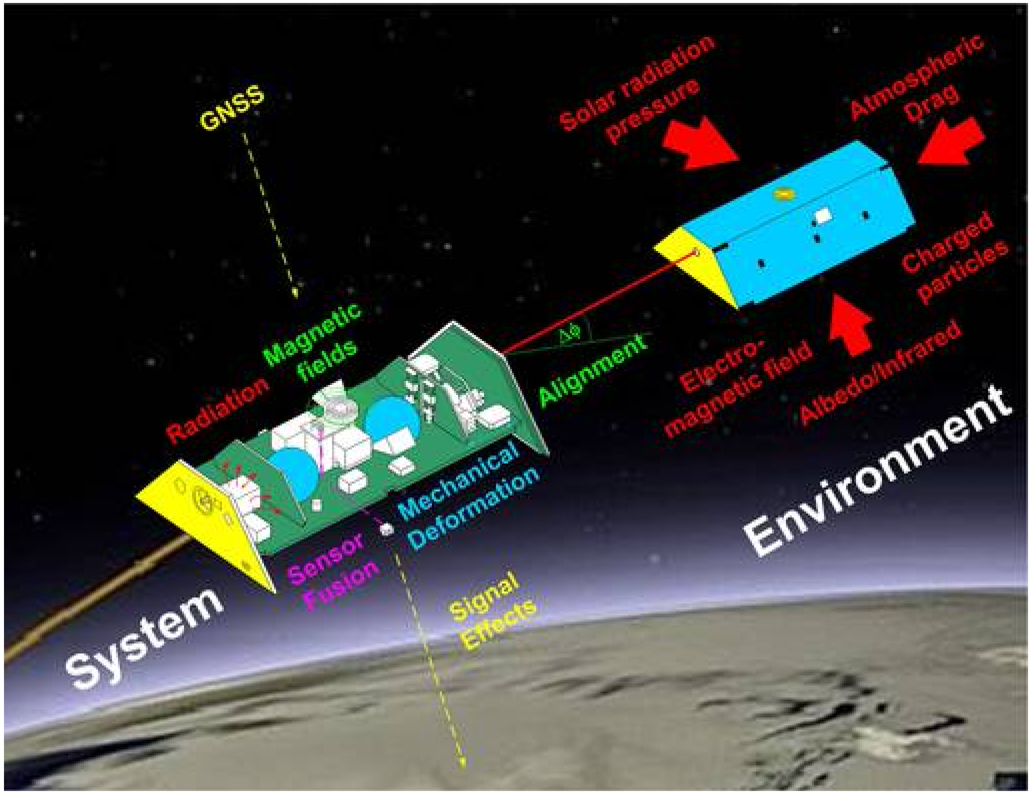
From Pool and Schmidt (1997), USGS Water Resources Investigations Report 97-4125

Example from USGS survey of groundwater-surface water interaction in Tucson, AZ.

GRACE Overview (1/3)

- As density of material varies below satellites, their angular velocity and radial distance from the center of the earth changes.

- GRACE and related missions perform high precision measurements of these displacements.



- Orbits have to be corrected for instrument noise and ‘collisions.’

- Time-lapse measurements can image changes in mass in areas not less than 150,000 km^2.

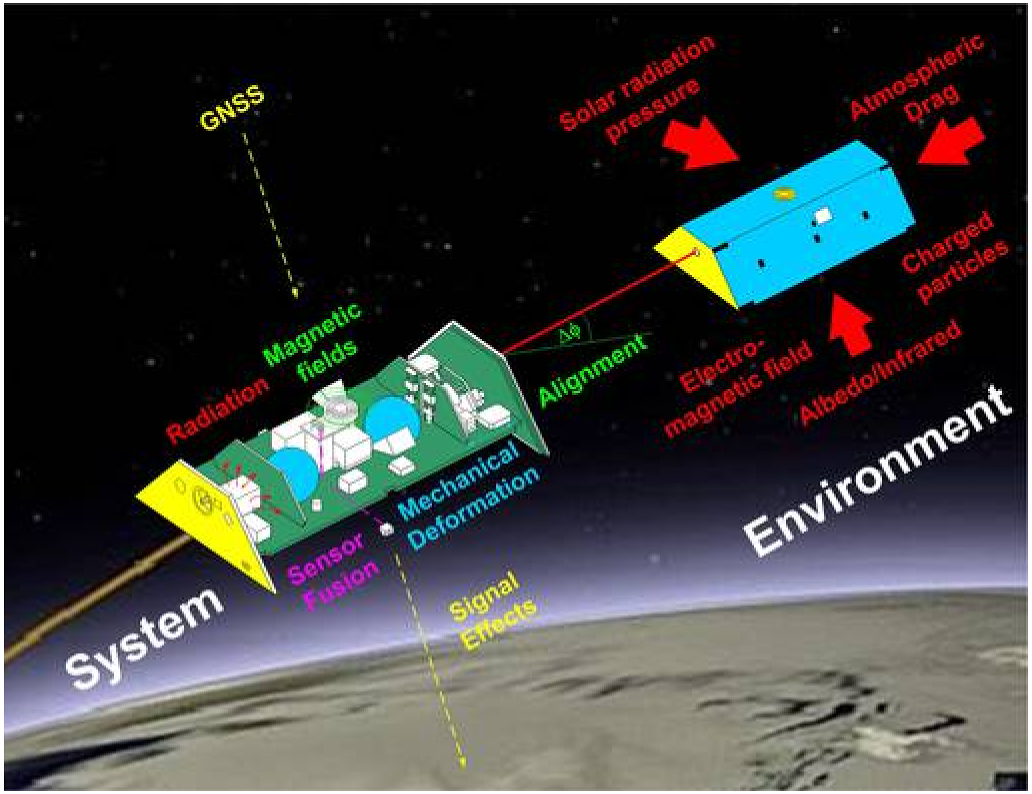
- Atmospheric mass must be corrected for.

https://www.zarm.uni-bremen.de/research/space-science/micro-satellite-systems-and-modelling-methods/research-areas/satellite-system-models.html

GRACE Overview (2/3)

- Time lapse measurements, so requires more than 1 overflight.

- Because of $1/r^2$ and the integral, measurements are most sensitive to local effects (atmosphere, etc.), which must be modeled out.



- Also because of $1/r^2$ dependence, the sampled areas are large, but measurements can still very precisely estimate the change of mass of the the large system.

https://www.zarm.uni-bremen.de/research/space-science/micro-satellite-systems-and-modelling-methods/research-areas/satellite-system-models.html

GRACE Overview (3/3)

- Joint project of Deutches Zentrum fuer Luft- and Raumfart (DLR) and NASA.
- Operational and analysis centers at UT-Austin’s Center for Space Research, NASA’s Jet Propulsion Laboratory, and the German Research Center for the Geoscience (GFZ).
- JPL produces (quasi-) gridded estimates of the mean and static gravitational field. GFZ produces a more complicated product that fits the field with spherical harmonic functions.
- Both correct for effects in space, but do not include atmospheric effects.
- CU-Boulder and Grace Tellus both produce products that correct for atmospheric effects, and correctly smooth the data.

https://www.zarm.uni-bremen.de/research/space-science/micro-satellite-systems-and-modelling-methods/research-areas/satellite-system-models.html

GRACE Hydrology Application—NW India

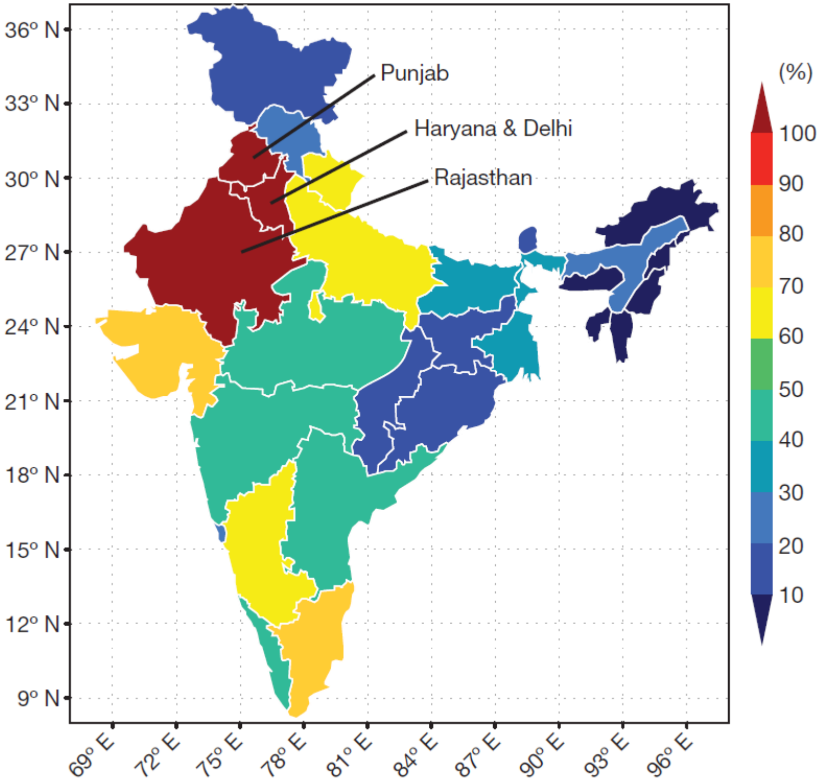


Figure 1 | Groundwater withdrawals as a percentage of recharge. The map is based on state-level estimates of annual withdrawals and recharge reported by the Indian Ministry of Water Resources[2]. The three states studied here are labelled.

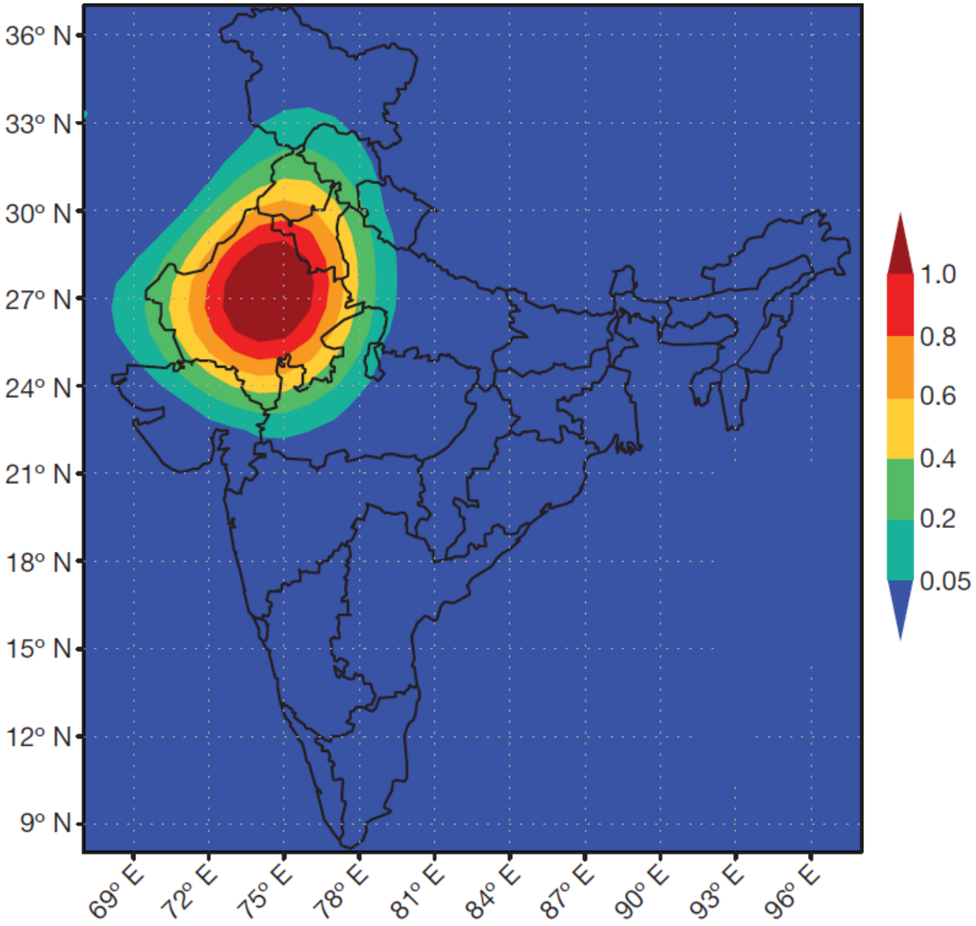


Figure 2 | GRACE averaging function. The unscaled, dimensionless averaging function used to estimate terrestrial water storage changes from GRACE data is mapped.

Rodell and others (2009, Nature)

GRACE Hydrology Application—NW India

- In general, must model changes in
    - surface water storage,
    - atmospheric mass,
    - reservoir storage,
    - soil moisture storage, and
    - snowpack storage.

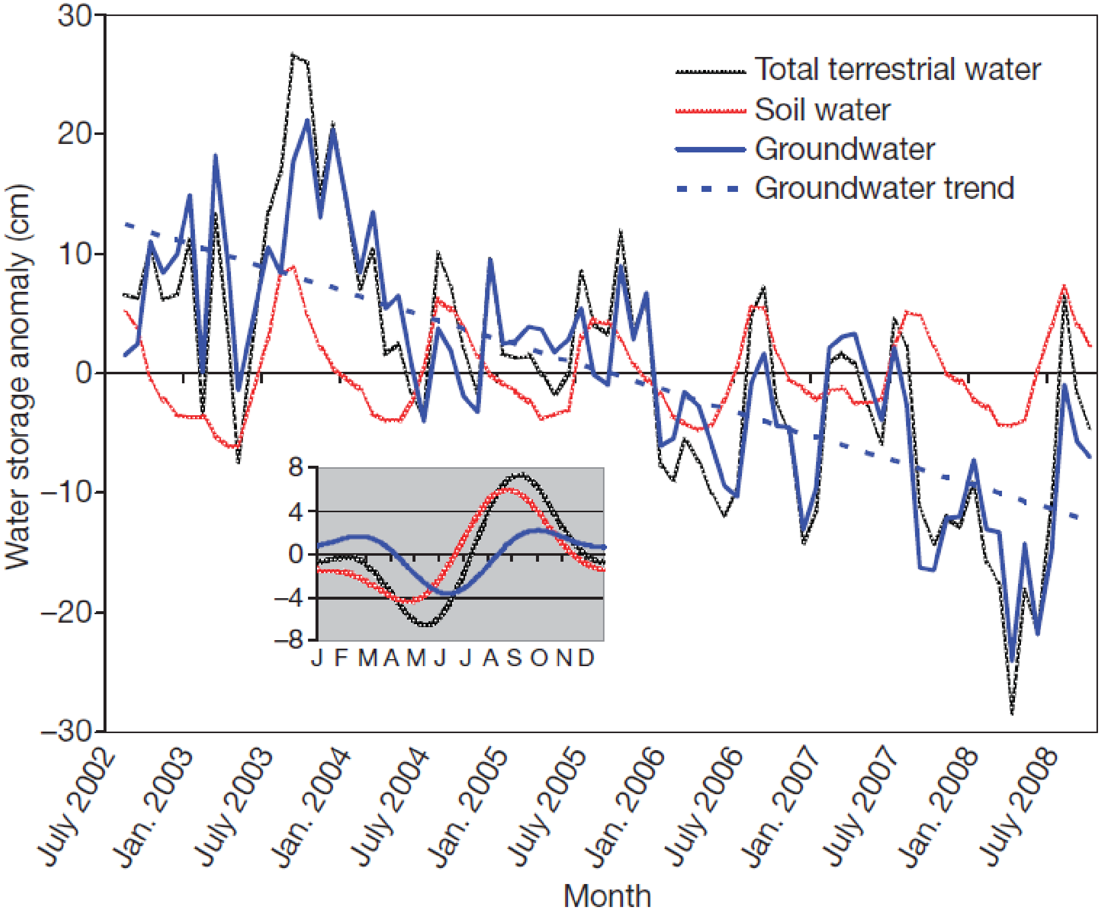


Figure 3 | Monthly time series of water storage anomalies in northwestern India. Monthly time series of anomalies of GRACE-derived total TWS, modelled soil-water storage and estimated groundwater storage, averaged over Rajasthan, Punjab and Haryana, plotted as equivalent heights of water in centimetres. Also shown is the best-fit linear groundwater trend. Inset, mean seasonal cycle of each variable.

Rodell and others (2009, Nature)