# ERTH 491-01 / GEOP 572-02
# Geodetic Methods

## – Lecture 20: Modeling - Strain 2 & Example –

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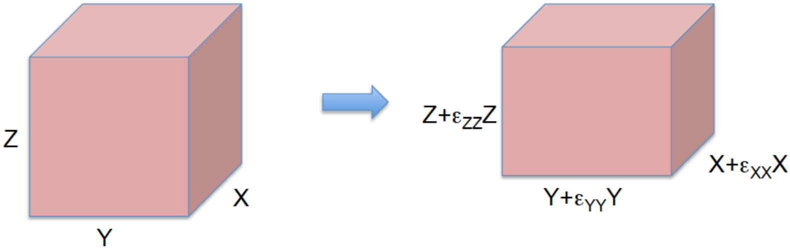
# Strain: Normal Strain

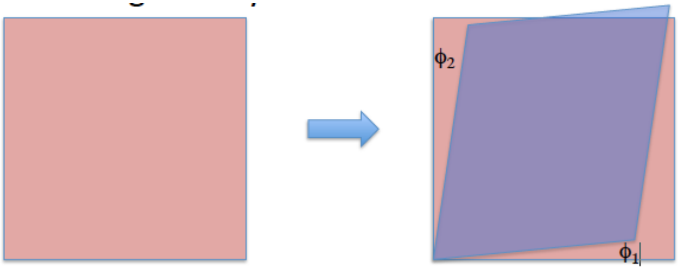



*J. Freymueller*

fractional length changes are **normal strains**:

$$\begin{aligned} \partial u_1/\partial x &= \varepsilon_{xx} \\ \partial u_2/\partial y &= \varepsilon_{yy} \\ \partial u_3/\partial z &= \varepsilon_{zz} \end{aligned}$$

components of strain proportional to derivatives of displacements in respective directions

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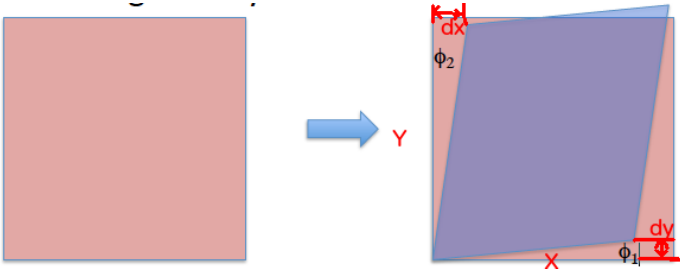
**shear components of strain** measure change in shape / angles

$$\varepsilon_{xy} = \varepsilon_{yx} = -\frac{1}{2}(\phi_1 + \phi_2)$$

# Strain: Shear Strain

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**shear components of strain** measure change in shape / angles

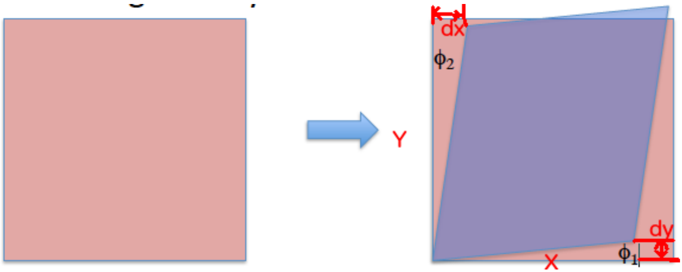
$$\varepsilon_{xy} = \varepsilon_{yx} = -\frac{1}{2}(\phi_1 + \phi_2)$$

angles are related to displacements:

$$tan(\phi_1) = \phi_1 = -\frac{dy}{X}$$

$$tan(\phi_2) = \phi_2 = -\frac{dx}{Y}$$

# Strain: Shear Strain

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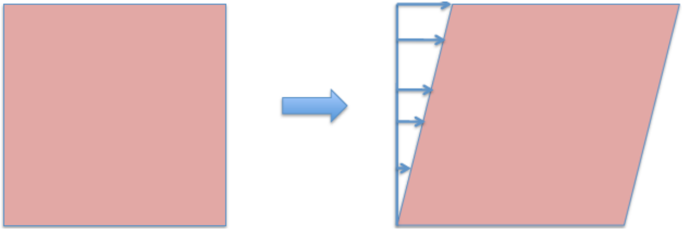
**shear components of strain** measure change in shape / angles

$$\varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x} + \frac{\partial u_1}{\partial y}\right)$$

subscripts: 1st – direction normal to element, 2nd – direction of shear

# Strain: Shear Strain



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**shear strain** results in solid body rotation if $\phi_1 \neq \phi_2$:

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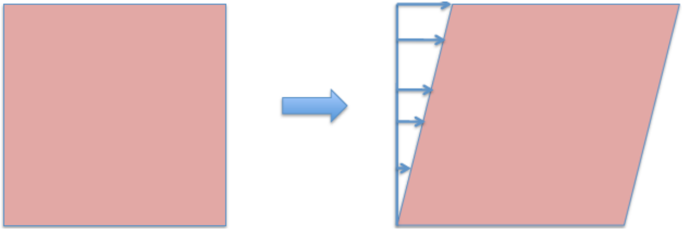

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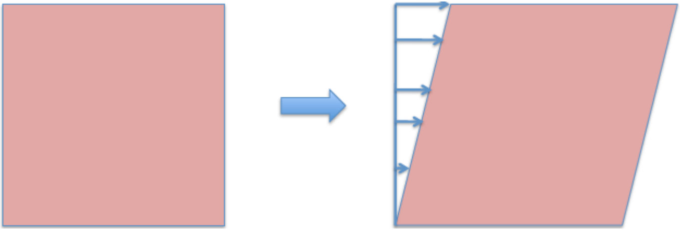
**shear strain** results in solid body rotation if $\phi_1 \neq \phi_2$ :

$$\omega_z = -\frac{1}{2}(\phi_1 - \phi_2) = \frac{1}{2}\left(\frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y}\right)$$

# Strain: Shear Strain



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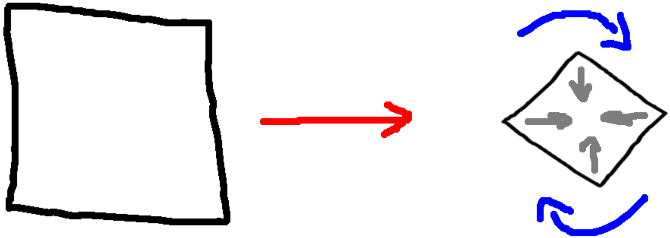
- if $\phi_1 = \phi_2$: no solid body rotation – **pure shear**
- if $\phi_1 = 0$: solid body rotation + shear – **simple shear** (strike slip faulting)

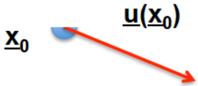
# Putting it all together



$$\begin{aligned} \textit{displacement} &= \textit{translation} + \textit{dilatation} + \textit{rotation} \\ u &\approx x + dx + \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right) + \frac{1}{2}\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right) \end{aligned}$$

correct formal description follows ...

# Displacement

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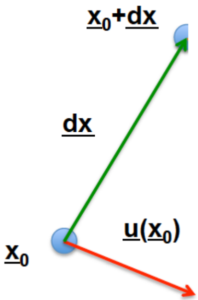


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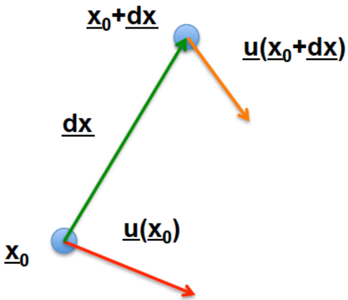


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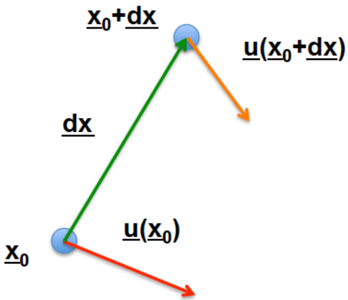


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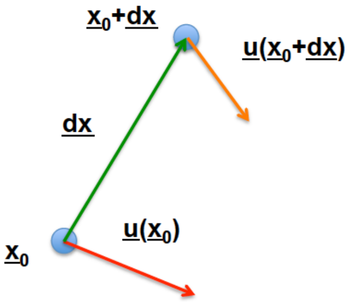
use Taylor Series expansion to relate the two vectors:

$$u_i(\mathbf{x_0} + \mathbf{dx}) \quad = \quad u_i(\mathbf{x_0}) + \left(\frac{\partial u_i}{\partial x_1}\right) dx_1 + \left(\frac{\partial u_i}{\partial x_2}\right) dx_2 + \left(\frac{\partial u_i}{\partial x_3}\right) dx_3$$

# Displacement

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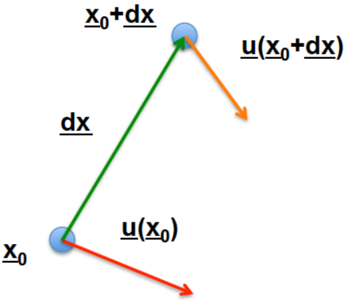
3 equations: i=1,2,3

first term: translation, remainder: rotation + dilatation

9 values $\partial u_i / \partial x_j$ for $i, j = 1 \ldots 3$

# Deformation Tensor




$$u(\mathbf{x_0} + \mathbf{dx}) = u(\mathbf{x_0}) + \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{bmatrix}$$

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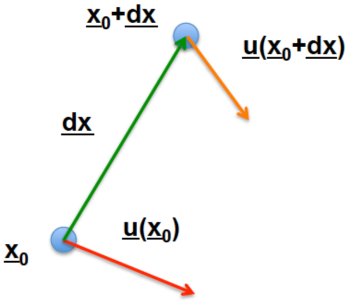


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- matrix describes dilatation and rotation
- is a 2-direction (rank 2) tensor: contains normal strain, and strain perpendicular to face on which it acts
- think of tensors as extension of vectors (magnitude and direction), which are an extension of scalars (magnitude)

# Separate Rotation and Strain

We can separate gradient tensor into the sum of two tensors: strain tensor and rotation tensor from:

$$\textit{displacement} = \textit{translation} + \textit{strain} + \textit{rotation}$$

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u_i(\mathbf{x_0} + \mathbf{dx}) &= u_i(\mathbf{x_0}) + \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) dx_j + \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) dx_j
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$$\begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) \\ \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) & \frac{\partial u_3}{\partial x_3} \end{bmatrix} +$$

$$\begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right) & 0 & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} - \frac{\partial u_3}{\partial x_2}\right) \\ \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1} - \frac{\partial u_1}{\partial x_3}\right) & \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right) & 0 \end{bmatrix}$$

rotation is anti-symmetric (see rotation matrix), strain part is symmetric

# Strain and Rotation Tensors

**Strain tensor** can be written:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

symmetric, with 6 independent components since
$\varepsilon_{21} = \varepsilon_{12}, \varepsilon_{31} = \varepsilon_{13}, \varepsilon_{32} = \varepsilon_{23}$

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**Rotation tensor** can be written:

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) = \begin{bmatrix} 0 & \omega_{12} & \omega_{13} \\ -\omega_{12} & 0 & \omega_{23} \\ -\omega_{13} & -\omega_{23} & 0 \end{bmatrix}$$

antisymmetric, with 3 independent components

# Strain and Rotation from GPS Data

- Can estimate all components of strain and rotation tensors directly from GPS data
- Equations in terms of 6 independent strain tensor components and 3 independent rotation tensor components
- . . . or in terms of the 9 components of the displacement gradient tensor

# Strain and Rotation from GPS Data

- Can estimate all components of strain and rotation tensors directly from GPS data
- Equations in terms of 6 independent strain tensor components and 3 independent rotation tensor components
- ... or in terms of the 9 components of the displacement gradient tensor
- Write motions relative to reference site or reference point in terms of distance from reference ("remove translation"):

$$u_i(\mathbf{x_0} + \mathbf{dx_0}) - u_i(\mathbf{x_0}) \quad = \quad \varepsilon_{ij} dx_j + \omega_{ij} dx_j$$

- $\mathbf{x_0}$ is reference location, **dx** is vector from reference to data location

# 2D Equations

- Again, assume infinitesimal displacements
- Velocity $v$ as function of position $\mathbf{p}$ can be expanded into:

$$v(\mathbf{x} + \mathbf{dx}) = v(\mathbf{x}) + \frac{\partial v}{\partial \mathbf{x}}\mathbf{dx}$$

- with

$$\begin{aligned} v_x(\mathbf{x} + \mathbf{dx}) &= v_x(\mathbf{x}) + \frac{\partial v_x}{\partial x}dx + \frac{\partial v_x}{\partial y}dy \\ v_y(\mathbf{x} + \mathbf{dx}) &= v_y(\mathbf{x}) + \frac{\partial v_y}{\partial x}dx + \frac{\partial v_y}{\partial y}dy \end{aligned}$$

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- So:

$$v(\mathbf{x} + \mathbf{dx}) = v(\mathbf{x}) + \nabla V \cdot \mathbf{dx}$$

- Where $\nabla V$ is velocity gradient tensor:

$$\nabla V = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} \end{bmatrix}$$

# 2D Equations

- Assuming 2 (GPS) sites separated by (small) distance **dx**:

$$\begin{aligned} v(\mathbf{x}+\mathbf{dx}) &= v(\mathbf{x}) + \nabla V \cdot \mathbf{dx} \\ v(\mathbf{x}+\mathbf{dx}) - v(\mathbf{x}) &= \nabla V \cdot \mathbf{dx} \\ v_i - v_j &= \nabla V \cdot \mathbf{dx} \end{aligned}$$

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\end{aligned}
$$

- Expand to 3 sites, where 1 site is reference:

$$
\begin{aligned}
\mathbf{d} &= G\mathbf{m} \\
velocity\_diff &= position\_differences \cdot velocity\_gradients
\end{aligned}
$$

$$
\begin{bmatrix} v_{x_2} - v_{x_1} \\ v_{y_2} - v_{y_1} \\ v_{x_3} - v_{x_1} \\ v_{y_3} - v_{y_1} \end{bmatrix} = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & 0 & 0 \\ 0 & 0 & x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 & 0 & 0 \\ 0 & 0 & x_3 - x_1 & y_3 - y_1 \end{bmatrix} \cdot \begin{bmatrix} \partial v_x / \partial x \\ \partial v_x / \partial y \\ \partial v_y / \partial x \\ \partial v_y / \partial y \end{bmatrix}
$$

- Solve for velocity gradients via least squares

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&= ??? \\
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&= \begin{bmatrix} \dot{\varepsilon}_{11} & \dot{\varepsilon}_{12} \\ \dot{\varepsilon}_{21} & \dot{\varepsilon}_{22} \end{bmatrix} + \begin{bmatrix} 0 & \dot{\omega} \\ -\dot{\omega} & 0 \end{bmatrix}
\end{aligned}
$$

# Now: Strain Rates and Rotation Rates

- we're doing this, because we deal with velocities a lot

$$\nabla V = \begin{bmatrix} \dot{\varepsilon}_{11} & \dot{\varepsilon}_{12} \\ \dot{\varepsilon}_{21} & \dot{\varepsilon}_{22} \end{bmatrix} + \begin{bmatrix} 0 & \dot{\omega} \\ -\dot{\omega} & 0 \end{bmatrix}$$

- Recall:

$$v(\mathbf{x} + \mathbf{dx}) = v(\mathbf{x}) + \nabla V \cdot \mathbf{dx}$$

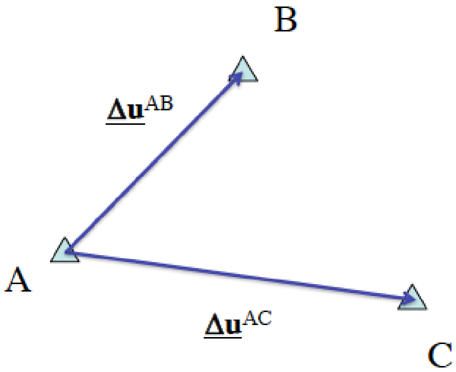
- 2D case:

$$\begin{aligned} v_x &= V_x + \dot{\varepsilon}_{xx}\Delta x + \dot{\varepsilon}_{xy}\Delta y + \dot{\omega}\Delta y \\ v_y &= V_y + \dot{\varepsilon}_{xy}\Delta x + \dot{\varepsilon}_{yy}\Delta y - \dot{\omega}\Delta x \end{aligned}$$

- 2D case: 4 parameters to solve for (3 strain, 1 rotation), need $\geq 2$ sites with horizontal data
- 3D case: 9 parameters to solve for (6 strain, 3 rotation), need $\geq 3$ sites with 3D data
- keep in mind that this applies to displacements, too ("drop the rate")

# Example: Strain from 3 GPS sites

- simple, general way to calculate average strain and rotation from 3 GPS sites
- (average strain for the area enclosed by the 3 sites)
- with more than 3 sites: divide network into triangles
- Delaunay triangulation implemented in GMT is a quick way to do so

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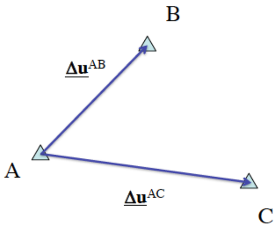


Let's look at this for a single baseline:

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$$\begin{bmatrix} \Delta u_1^{AB} \\ \Delta u_2^{AB} \end{bmatrix} = \begin{bmatrix} \varepsilon_{11}\Delta x_1^{AB} + \varepsilon_{12}\Delta x_2^{AB} + \omega_{12}\Delta x_2^{AB} \\ \varepsilon_{12}\Delta x_1^{AB} + \varepsilon_{22}\Delta x_2^{AB} - \omega_{12}\Delta x_1^{AB} \end{bmatrix}$$

$$=$$

$$= G \cdot \mathbf{m}$$

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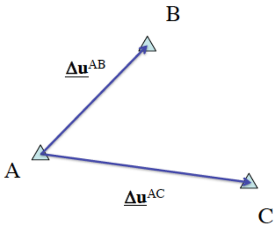


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$$= \begin{bmatrix} \quad \end{bmatrix} \cdot \begin{bmatrix} \quad \end{bmatrix}$$

$$= G \cdot \mathbf{m}$$

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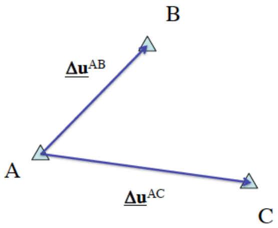


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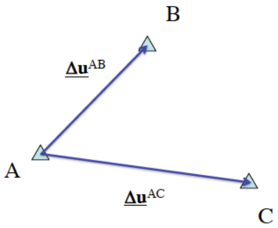
*J. Freymueller*

$$\begin{bmatrix} \Delta u_1^{AB} \\ \Delta u_2^{AB} \end{bmatrix} = \begin{bmatrix} \varepsilon_{11}\Delta x_1^{AB} + \varepsilon_{12}\Delta x_2^{AB} + \omega_{12}\Delta x_2^{AB} \\ \varepsilon_{12}\Delta x_1^{AB} + \varepsilon_{22}\Delta x_2^{AB} - \omega_{12}\Delta x_1^{AB} \end{bmatrix}$$

$$= \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{22} \\ \omega_{12} \end{bmatrix}$$

$$= G \cdot \mathbf{m}$$

# Example: Strain from 3 GPS sites




Let's look at this for a single baseline:

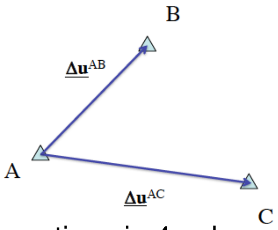
*J. Freymueller*

$$\begin{bmatrix} \Delta u_1^{AB} \\ \Delta u_2^{AB} \end{bmatrix} = \begin{bmatrix} \varepsilon_{11}\Delta x_1^{AB} + \varepsilon_{12}\Delta x_2^{AB} + \omega_{12}\Delta x_2^{AB} \\ \varepsilon_{12}\Delta x_1^{AB} + \varepsilon_{22}\Delta x_2^{AB} - \omega_{12}\Delta x_1^{AB} \end{bmatrix}$$

$$= \begin{bmatrix} \Delta x_1^{AB} & \Delta x_2^{AB} & 0 & \Delta x_2^{AB} \\ 0 & \Delta x_1^{AB} & \Delta x_2^{AB} & -\Delta x_1^{AB} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{22} \\ \omega_{12} \end{bmatrix}$$

$$= G \cdot \mathbf{m}$$

# Example: Strain from 3 GPS sites

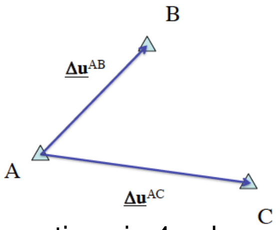



Using all sites we get 4 equations in 4 unknowns:

*J. Freymueller*

$$\begin{bmatrix} \Delta u_1^{AB} \\ \Delta u_2^{AB} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \Delta x_1^{AB} & \Delta x_2^{AB} & 0 & \Delta x_2^{AB} \\ 0 & \Delta x_1^{AB} & \Delta x_2^{AB} & -\Delta x_1^{AB} \\ & & & \\ & & & \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{22} \\ \omega_{12} \end{bmatrix}$$

# Example: Strain from 3 GPS sites

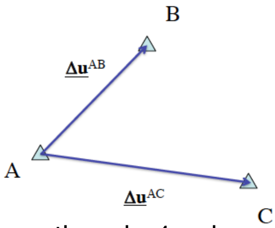



Using all sites we get 4 equations in 4 unknowns:

*J. Freymueller*

$$
\begin{bmatrix} \Delta u_1^{AB} \\ \Delta u_2^{AB} \\ \Delta u_1^{AC} \\ \Delta u_2^{AC} \end{bmatrix} = \begin{bmatrix} \Delta x_1^{AB} & \Delta x_2^{AB} & 0 & \Delta x_2^{AB} \\ 0 & \Delta x_1^{AB} & \Delta x_2^{AB} & -\Delta x_1^{AB} \\ \Delta x_1^{AC} & \Delta x_2^{AC} & 0 & \Delta x_2^{AC} \\ 0 & \Delta x_1^{AC} & \Delta x_2^{BC} & -\Delta x_1^{AC} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{22} \\ \omega_{12} \end{bmatrix}
$$

# Example: Strain from 3 GPS sites




Using all sites we get 4 equations in 4 unknowns:

*J. Freymueller*

$$\begin{bmatrix} \Delta u_1^{AB} \\ \Delta u_2^{AB} \\ \Delta u_1^{AC} \\ \Delta u_2^{AC} \end{bmatrix} = \begin{bmatrix} \Delta x_1^{AB} & \Delta x_2^{AB} & 0 & \Delta x_2^{AB} \\ 0 & \Delta x_1^{AB} & \Delta x_2^{AB} & -\Delta x_1^{AB} \\ \Delta x_1^{AC} & \Delta x_2^{AC} & 0 & \Delta x_2^{AC} \\ 0 & \Delta x_1^{AC} & \Delta x_2^{BC} & -\Delta x_1^{AC} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{22} \\ \omega_{12} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{22} \\ \omega_{12} \end{bmatrix} = \begin{bmatrix} \Delta x_1^{AB} & \Delta x_2^{AB} & 0 & \Delta x_2^{AB} \\ 0 & \Delta x_1^{AB} & \Delta x_2^{AB} & -\Delta x_1^{AB} \\ \Delta x_1^{AC} & \Delta x_2^{AC} & 0 & \Delta x_2^{AC} \\ 0 & \Delta x_1^{AC} & \Delta x_2^{AC} & -\Delta x_1^{AC} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Delta u_1^{AB} \\ \Delta u_2^{AB} \\ \Delta u_1^{AC} \\ \Delta u_2^{AC} \end{bmatrix}$$