ERTH 491-01 / GEOP 572-02
Geodetic Methods

– Lecture 22: Modeling - Strain Examples –

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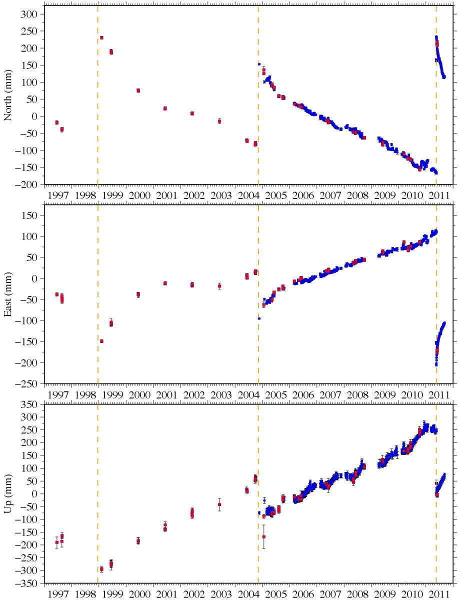
November 02, 2015

Guess The Process . . .



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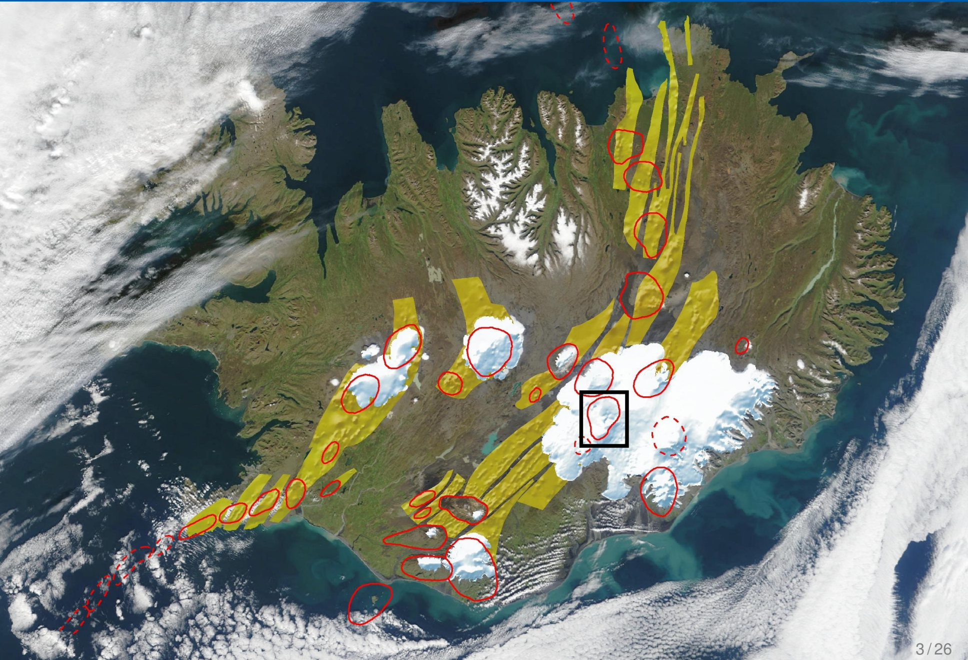

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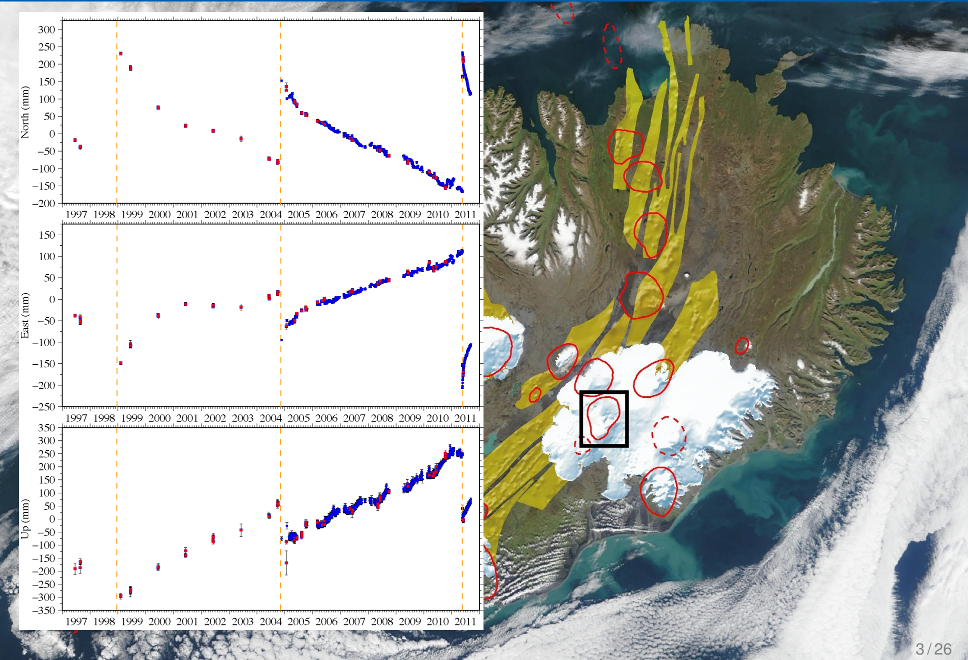

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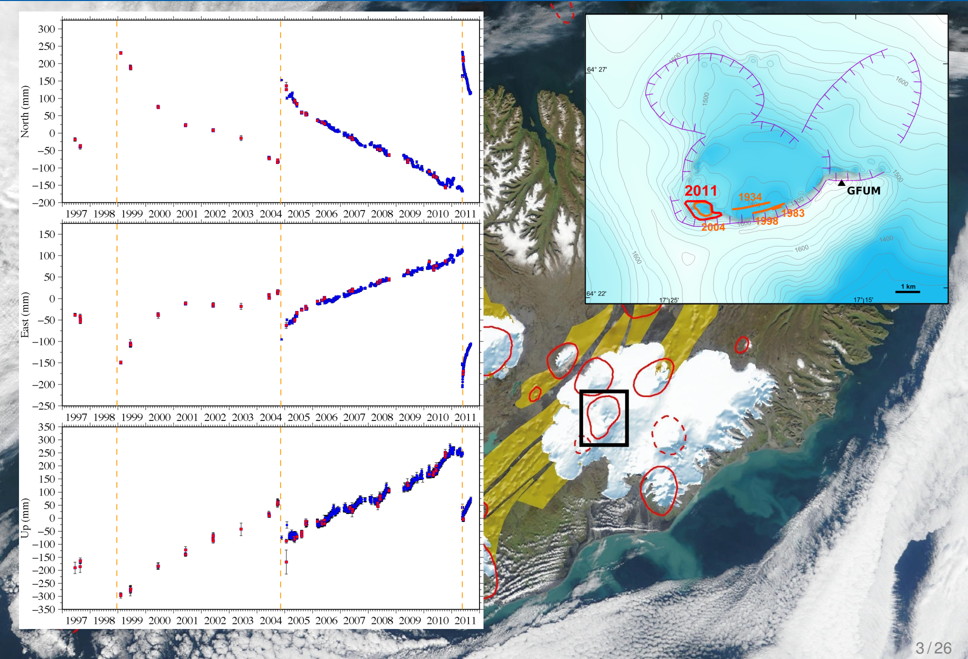

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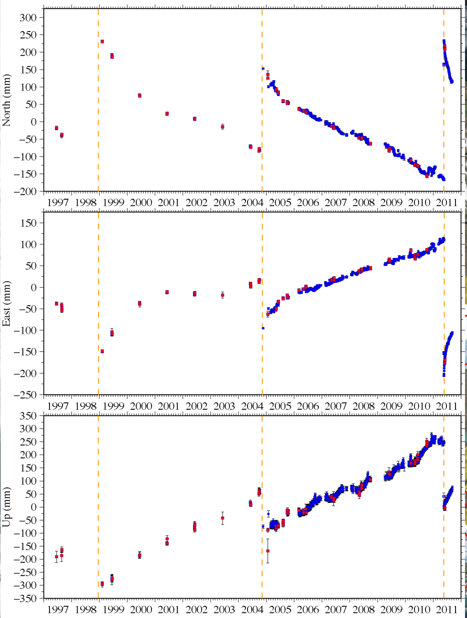

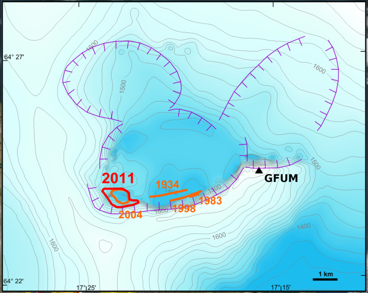

- explosive eruption 21-28 May 2011
- plumes > 20 *km*
- continuous inflation, gradual increase in seismicity

2D Equations

- Again, assume infinitesimal displacements
- Velocity v as function of position $\mathbf{p}$ can be expanded into:

$$v(\mathbf{x} + \mathbf{dx}) = v(\mathbf{x}) + \frac{\partial v}{\partial \mathbf{x}} \mathbf{dx}$$

- with

$$\begin{aligned} v_x(\mathbf{x} + \mathbf{dx}) &= v_x(\mathbf{x}) + \frac{\partial v_x}{\partial x} dx + \frac{\partial v_x}{\partial y} dy \\ v_y(\mathbf{x} + \mathbf{dx}) &= v_y(\mathbf{x}) + \frac{\partial v_y}{\partial x} dx + \frac{\partial v_y}{\partial y} dy \end{aligned}$$

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- So:

$$v(\mathbf{x} + \mathbf{dx}) \quad = \quad v(\mathbf{x}) + \nabla V \cdot \mathbf{dx}$$

- Where ∇V is velocity gradient tensor:

$$\nabla V \quad = \quad \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} \end{bmatrix}$$

Strain Rates and Rotation Rates

- displacement contains strain and rigid body rotation
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&= ??? \\
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\end{aligned}
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&= \begin{bmatrix} \dot{\varepsilon}_{11} & \dot{\varepsilon}_{12} \\ \dot{\varepsilon}_{21} & \dot{\varepsilon}_{22} \end{bmatrix} + \begin{bmatrix} 0 & \dot{\omega} \\ -\dot{\omega} & 0 \end{bmatrix}
\end{aligned}
$$

Now: Strain Rates and Rotation Rates

- we're doing this, because we deal with velocities a lot

$$\nabla V = \begin{bmatrix} \dot{\varepsilon}_{11} & \dot{\varepsilon}_{12} \\ \dot{\varepsilon}_{21} & \dot{\varepsilon}_{22} \end{bmatrix} + \begin{bmatrix} 0 & \dot{\omega} \\ -\dot{\omega} & 0 \end{bmatrix}$$

- Recall:

$$v(\mathbf{x} + \mathbf{dx}) = v(\mathbf{x}) + \nabla V \cdot \mathbf{dx}$$

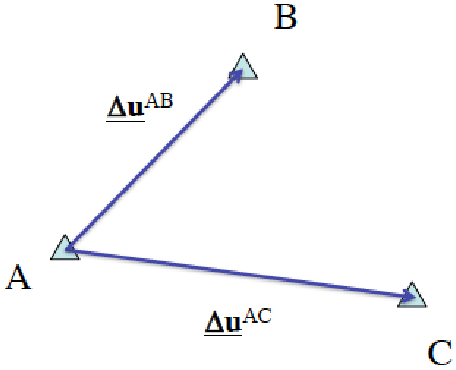
- 2D case:

$$\begin{aligned} v_x &= V_x + \dot{\varepsilon}_{xx}\Delta x + \dot{\varepsilon}_{xy}\Delta y + \dot{\omega}\Delta y \\ v_y &= V_y + \dot{\varepsilon}_{xy}\Delta x + \dot{\varepsilon}_{yy}\Delta y - \dot{\omega}\Delta x \end{aligned}$$

- 2D case: 4 parameters to solve for (3 strain, 1 rotation), need ≥ 2 sites with horizontal data
- 3D case: 9 parameters to solve for (6 strain, 3 rotation), need ≥ 3 sites with 3D data
- keep in mind that this applies to displacements, too ("drop the rate")

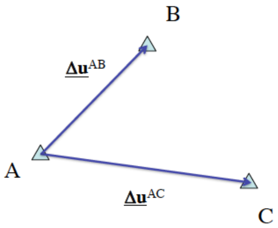
Example: Strain from 3 GPS sites

- simple, general way to calculate average strain and rotation from 3 GPS sites
- (average strain for the area enclosed by the 3 sites)
- with more than 3 sites: divide network into triangles
- Delaunay triangulation implemented in GMT is a quick way to do so



J. Freymueller

Example: Strain from 3 GPS sites



Let's look at this for a single baseline:

J. Freymueller

$$\begin{bmatrix} \Delta u_1^{AB} \\ \Delta u_2^{AB} \end{bmatrix} = \begin{bmatrix} \varepsilon_{11}\Delta x_1^{AB} + \varepsilon_{12}\Delta x_2^{AB} + \omega_{12}\Delta x_2^{AB} \\ \varepsilon_{12}\Delta x_1^{AB} + \varepsilon_{22}\Delta x_2^{AB} - \omega_{12}\Delta x_1^{AB} \end{bmatrix}$$

$$=$$

$$= G \cdot \mathbf{m}$$

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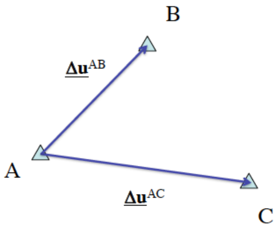

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$$= \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \cdot \begin{bmatrix} \quad \\ \quad \\ \quad \\ \quad \end{bmatrix}$$

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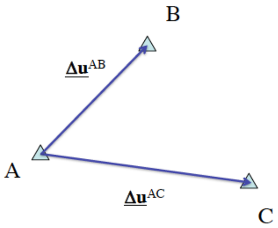

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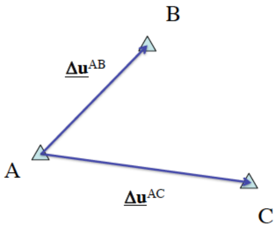

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$$

$$
= \begin{bmatrix} \Delta x_1^{AB} & \Delta x_2^{AB} & 0 & \Delta x_2^{AB} \\ 0 & \Delta x_1^{AB} & \Delta x_2^{AB} & -\Delta x_1^{AB} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{22} \\ \omega_{12} \end{bmatrix}
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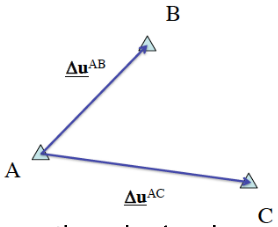

Using all sites we get 4 equations in 4 unknowns:

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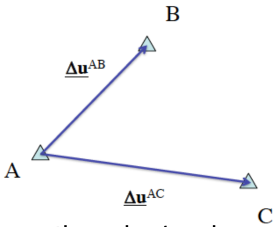

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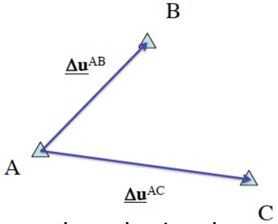

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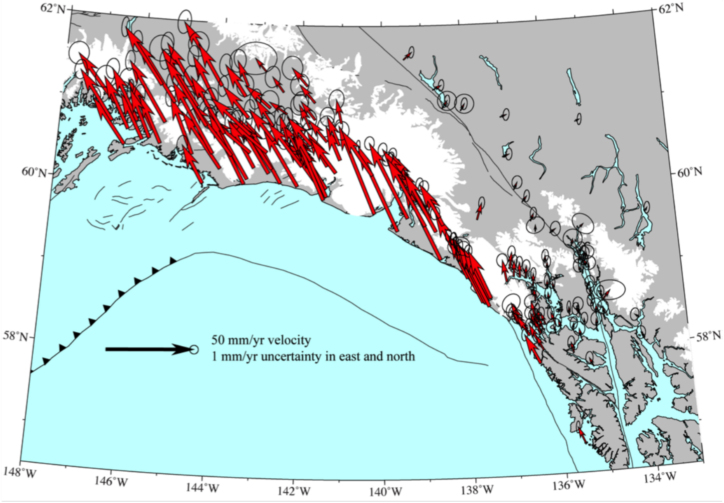
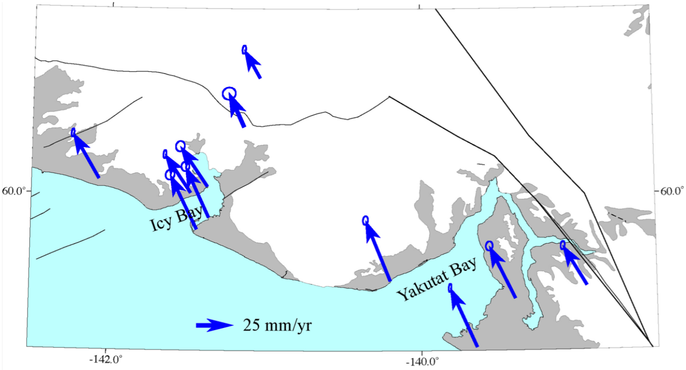
Example: SE Alaska



Julie Elliott

Example: SE Alaska, Icy Bay



Julie Elliott

- velocities relative to stable North America *(Sella et al, 2007)*
- velocities corrected for GIA using model of *Larson et al (2005)*

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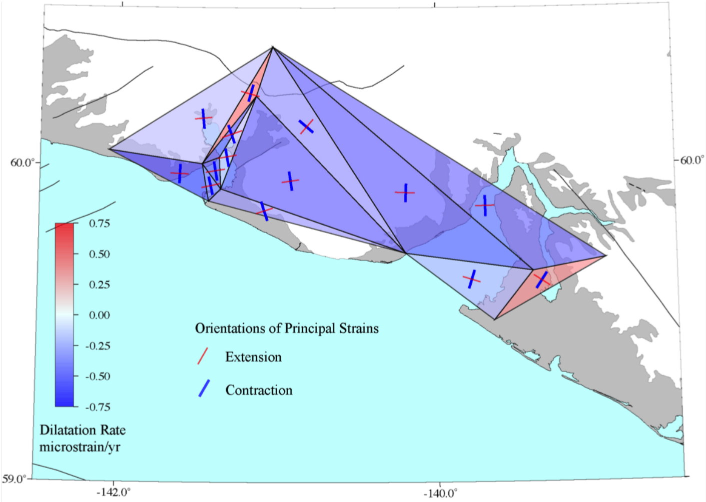

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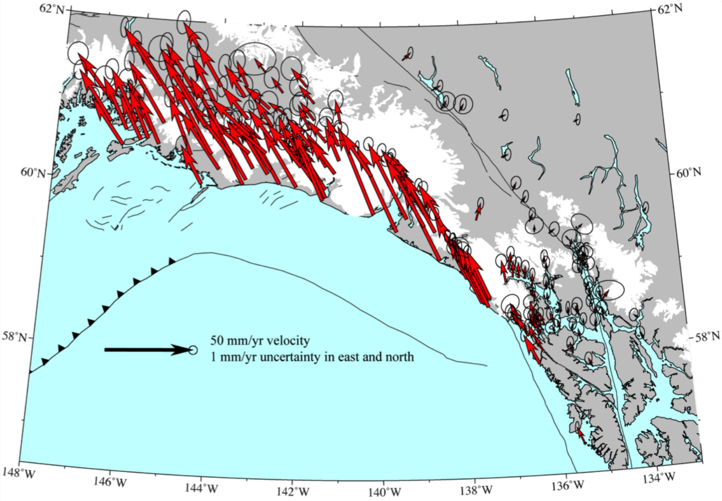

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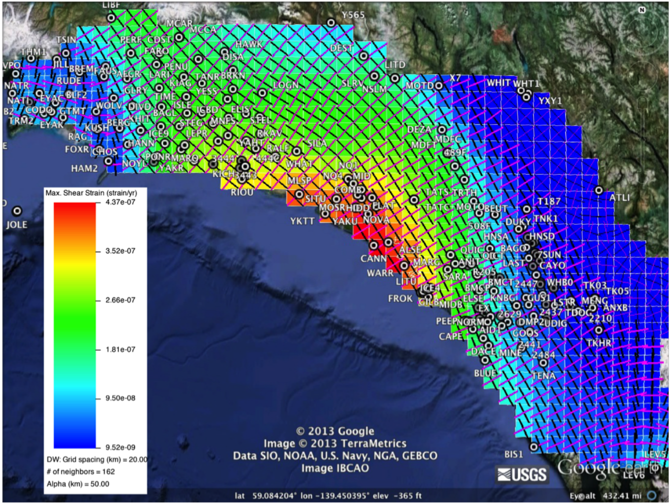

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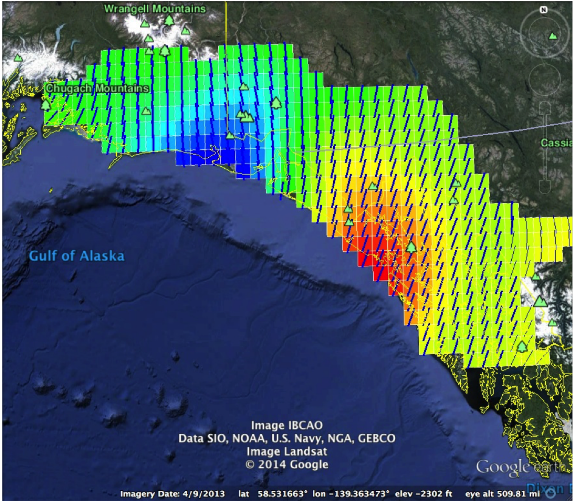

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Strain Varies over Space

- In most cases strain varies over space, can't assume that uniform strain exists over large area
- Strike-slip fault: strain varies with distance x from the fault (v: slip rate, D locking depth):

$$\dot{\varepsilon}_{12} = \frac{vD}{2\pi} \frac{1}{(x^2 + D^2)}$$

- Options:
  - Ignore variations and use uniform approximation
  - Fit mathematical model for ε and ω
  - Use sub-regions where you expect uniform strain (clustering?)
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  - Map strain by looking at each triangle
  - Map strain in more continuous way/gridded solution:
    - nearest neighbor: calculate velocity gradient for fixed number of nearby sites
    - distance weighted: use all stations in strain calculation, but weight data by distance from grid node
    - Haines and Holt: use splines to create interpolated velocity field; derive strain rates, vorticity rates and expected velocities from this.

Strain Varies over Space: Anatolia

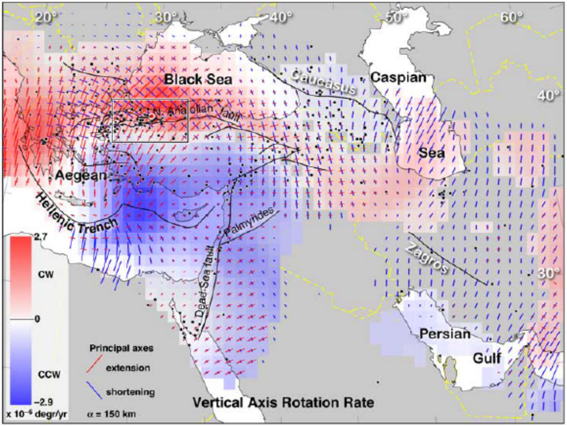

Figure 2. Map of GPS strain and rotation rates for the eastern Mediterranean/Middle East. The principal structures of the region are drawn with heavy black lines and labeled. Principal infinitesimal horizontal strain axes are shown with short colored line segments scaled by the absolute value of their magnitude; red are extension and blue are shortening. Colors in this plot show the variation in magnitude of rotation about a vertical, downward positive axis, with red indicating clockwise (CW) and blue indicating counterclockwise (CCW). Boxes are blank where the absolute value of the magnitude is less than the one sigma error. GPS vectors from *Reilinger et al.* [2006] are shown beneath the semitransparent colors. The red dot shows the epicenter of the 1999 *M*7.5 Izmit earthquake. Box shows the location of the coseismic strain plot shown in Figure 10.

Allmendinger et al., 2007

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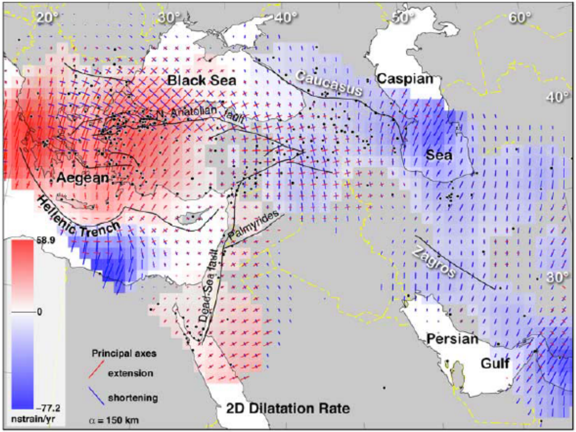

Figure 3. Map showing the magnitude of two-dimensional dilatation rate in the eastern Mediterranean/ Middle East.

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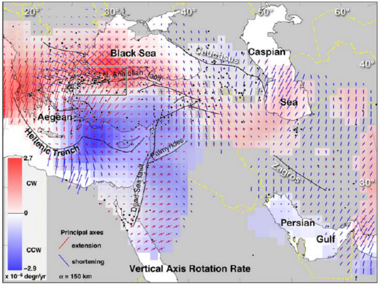

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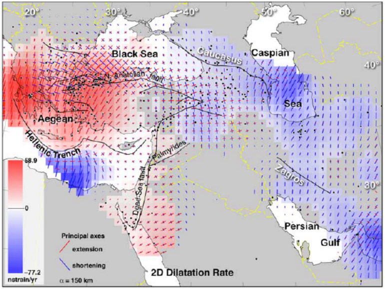

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Strain Varies over Space: Tibet

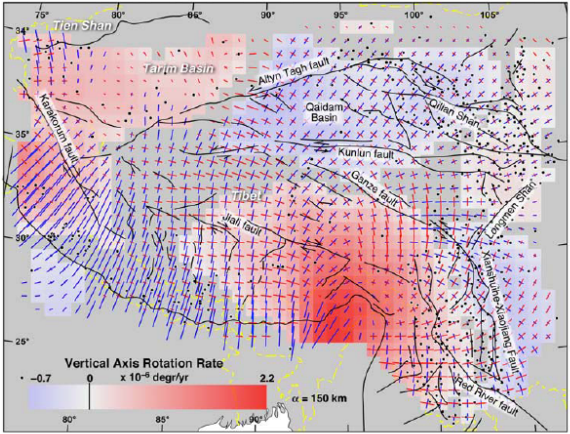

Figure 4. Map of GPS strain and rotation rates for Tibet and the Himalaya. GPS vectors are from *Zhang et al.* [2004]. Major structures are shown with heavy black lines. Other features are as described in Figure 2.

Allmendinger et al., 2007

Strain Varies over Space: Tibet

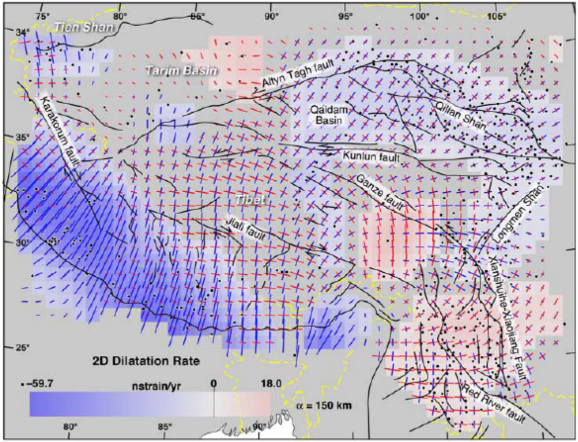

Figure 5. Map of the magnitude of two-dimensional dilatation rate in and around Tibet, calculated from the GPS vectors of *Zhang et al.* [2004]. Red is positive and blue is negative. Other features are as described in Figure 2.

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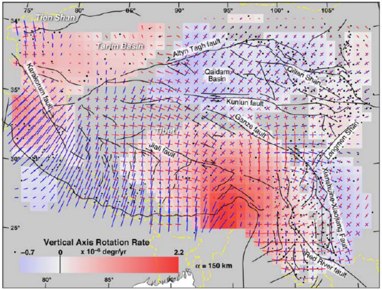
Strain Varies over Space: Tibet



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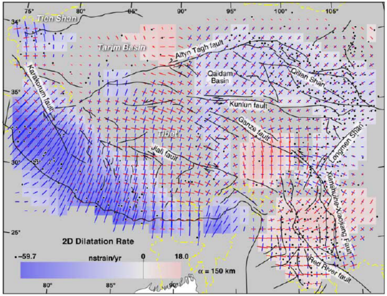

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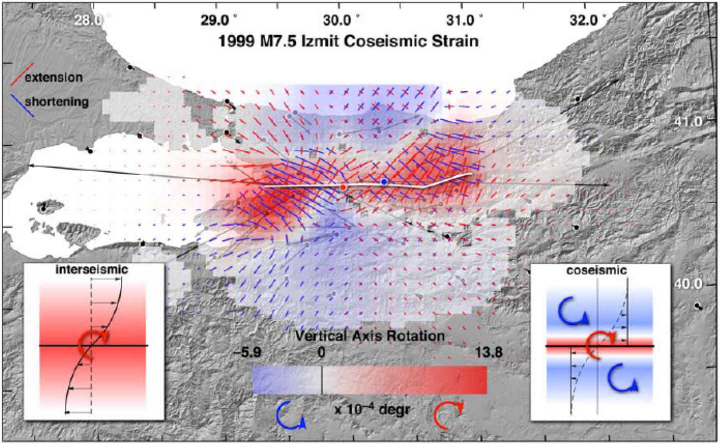

Figure 10. Coseismic vertical axis rotation rate and principal infinitesimal strain rate axes for the 1999 *M*7.5 Izmit earthquake. Analyzed GPS data are from *Reilinger et al.* [2000]. Heavy white line shows the general extent of the ruptured segment of the North Anatolian fault. Insets show the expected deformation patterns for interseismic elastic loading and coseismic elastic rebound.

Allmendinger et al., 2007

Strain Varies over Space: Global

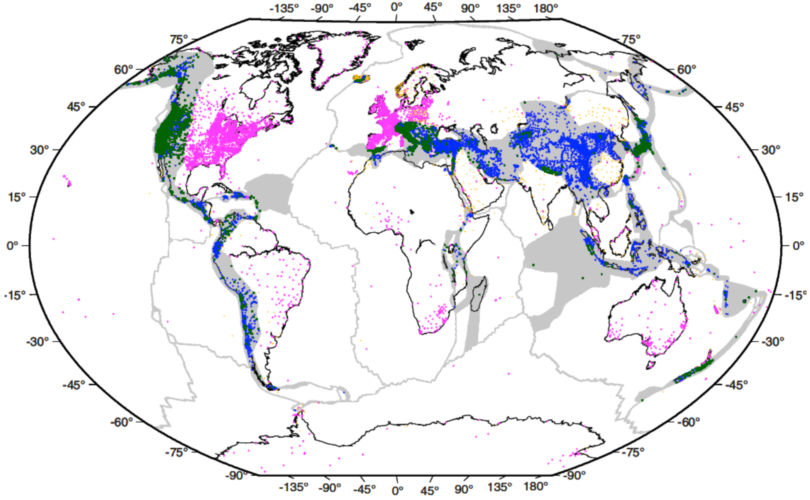

Figure 1. Gray shading is outline of all areas allowed to deform. White areas comprise of 50 assumed rigid plates. Purple and green dots are for GPS stations on rigid plates or in deforming zones, respectively, for which we derived a velocity. Yellow and blue dots are for GPS stations on rigid plates or in deforming zones, respectively, for which we took the velocity from the literature.

Kreemer et al., 2014

Strain Varies over Space: Global

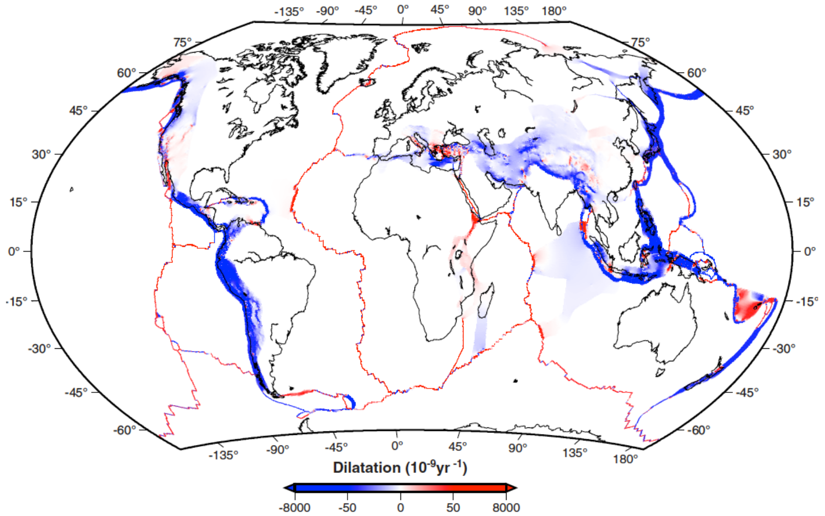

Figure 4. Dilatational component of the modeled strain rates ($\dot{\varepsilon}_{\varphi\varphi}+\dot{\varepsilon}_{\theta\theta}$). Extension (contraction) is positive (negative). Highest values are saturated.

Kreemer et al., 2014

Strain Varies over Space: Global

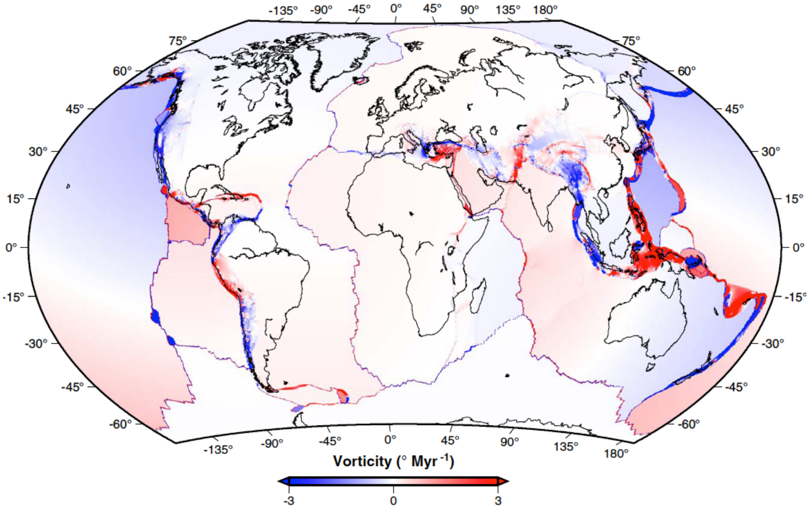

Figure 6. Vorticity associated with the velocity gradient tensor field (positive is counterclockwise). To also show the vorticity associated with rigid-body rotations, we chose the reference frame of *Kreemer* [2009] relative to the subasthenospheric mantle. Scale is saturated.

Kreemer et al., 2014