



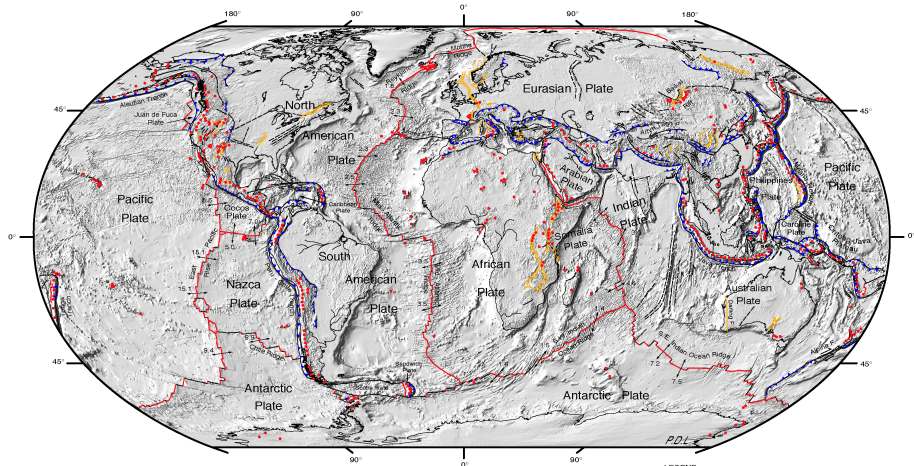
ERTH 491-01 / GEOP 572-02
Geodetic Methods

– Lecture 23: Modeling - Plate Kinematics –

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Tectonic Activity



DIGITAL TECTONIC ACTIVITY MAP OF THE EARTH
 Tectonism and Volcanism of the Last One Million Years
DTAM - 1



NASA/Goddard Space Flight Center
 Greenbelt, Maryland 20771

Robinson Projection
 October 2002

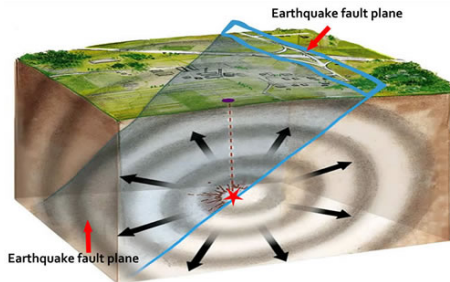
- LEGEND**
- Actively-spreading ridges and transform faults
 - Total spreading rate, cm/year
 - Major active fault or fault zone; dashed where nature, location, or actively uncertain
 - Normal fault or rift; hachures on downthrown side
 - Reverse fault (overthrust, subduction zones); generalized; bars on upthrown side
 - Volcanic centers active within the last one million years; generalized. Minor basaltic centers and seamounts omitted.

Plates and Boundaries

- plates are rigid, relative motions occur on their boundaries
- how many plates / microplates are there?
- plate boundaries have some finite width: plate boundary zones
 - can be narrow: < 10 km
 - or very wide: 500 – 1000 km
- relative motion occurs on faults, or breaks in the Earth's lithosphere

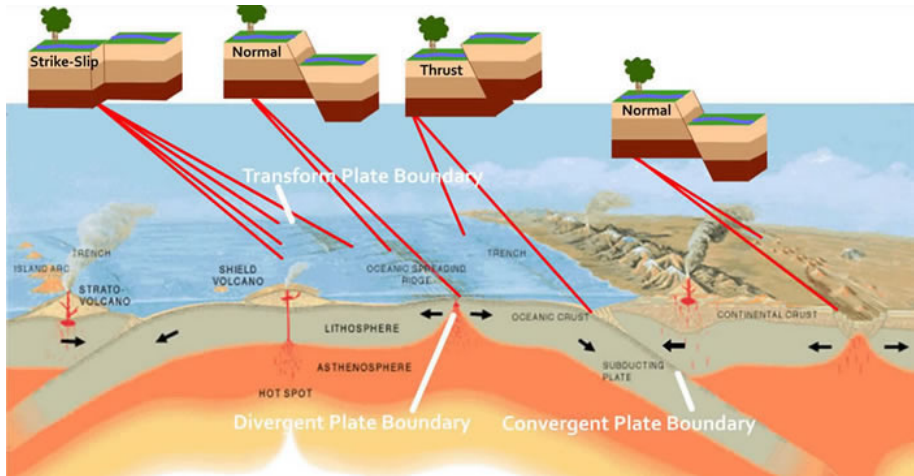
Faults

- Faults are (approximately) planar surfaces
- Motion on either side of the surface relative to the other
- Direction of motion is slip direction
- Motion driven by plate tectonics
- Nature of slip depends on depth:
 - shallow: fault stuck together (friction), slip occurs suddenly in earthquakes
 - deep: fault slips mostly at steady rate



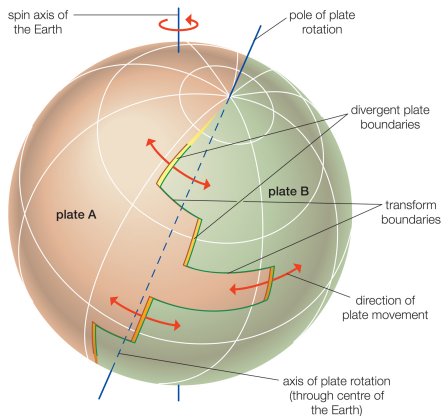
Valerie Thomas, USGS & Anthony Guarino, Caltech

Types of Faults



Valerie Thomas, USGS & Anthony Guarino, Caltech

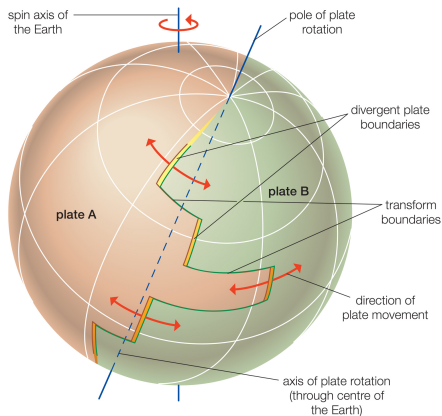
Motion on a sphere



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- rigid motion on sphere is about geometric axis
- 2 equivalent ways to describe:
 - pole of rotation (Euler Pole) and angular speed (deg/Myr)
 - angular velocity vector

Motion on a sphere



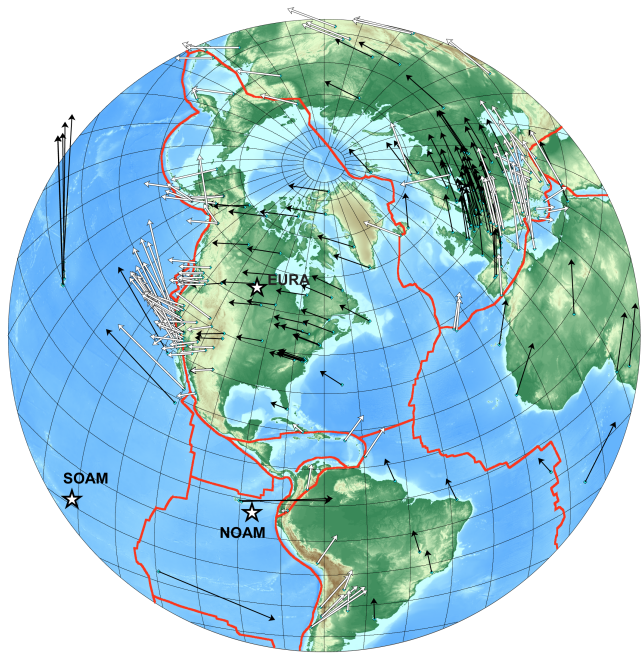
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- relative plate motion direction given by transform fault direction
- transform fault indicates small circle about rotation pole
- relative plate motion rate given by seafloor magnetic isochrons
- GPS velocities are direct measure of plate motion direction and magnitude
- GPS velocities are normal to great circle passing through pole of rotation

Geologic Plate Motion Models

- Relative plate motion models based on a combination of
 - Mid-ocean ridge spreading rate (from marine magnetic anomalies)
 - transform fault azimuths
 - earthquake slip vectors (hard)
- some plates have little or no data (Caribbean, Philippine Sea Plates)
- Common models: NUVEL-1, revised to NUVEL-1A
- New model: MORVEL (DeMets et al, 2010)

Motion on a sphere



courtesy: Jeff Freymueller, UAF

Conversion between Euler Parameters and Rotation Vector

- Rotation Vector: $\Omega(\omega_x, \omega_y, \omega_z)$
- Euler Parameters: latitude λ , longitude ϕ , angular speed s
- Euler to Rotation:

$$\omega_x = s \cos(\lambda) \cos(\phi)$$

$$\omega_y = s \cos(\lambda) \sin(\phi)$$

$$\omega_z = s \sin(\lambda)$$

- Rotation to Euler:

$$\lambda = \arctan \left(\frac{\omega_z}{\sqrt{\omega_x^2 + \omega_y^2}} \right)$$

$$\phi = \arctan \left(\frac{\omega_y}{\omega_x} \right)$$

$$s = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2}$$

Calculating Site Velocities

- Easiest to use angular velocity vector to compute site velocities
- Cross product of site location P (ECEF: $P = [X,Y,Z]$) with plate angular velocity:

$$\vec{v} = \vec{\omega} \times \vec{P}$$

- expand cross product to rewrite as matrix equation:

$$\begin{aligned}\vec{v} &= \vec{\omega} \times \vec{P} \\ &= (Z\omega_y - Y\omega_z)\hat{x} + (X\omega_z - Z\omega_x)\hat{y} + (Y\omega_x - X\omega_y)\hat{z}\end{aligned}$$

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- units: Ω usually in $^{\circ}/Myr$

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- want m/yr :
 - time-factor: 10^{-6}
 - degrees-to-radians-to-arclength: $\frac{\pi}{180} R$
 - where R is mean Earth radius: $R = 6,378,137$ m

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- \vec{v} in geocentric coordinates $[v_x, v_y, v_z]$ units of m/yr:

$$\vec{v} = 10^{-6} \frac{\pi}{180} R (\vec{\omega}_{deg/Myr} \times \vec{P})$$

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- need location conversion from geodetic to geocentric frame (lat,lon to X,Y,Z)
- need velocity conversion from geocentric to local frame (X,Y,Z to N,E,U)

Recall: Geodetic to Geocentric

$$X = (N + h)\cos(\lambda)\cos(\phi)$$

$$Y = (N + h)\cos(\lambda)\sin(\phi)$$

$$Z = \left(\frac{a^2}{b^2}N + h\right)\sin(\lambda)$$

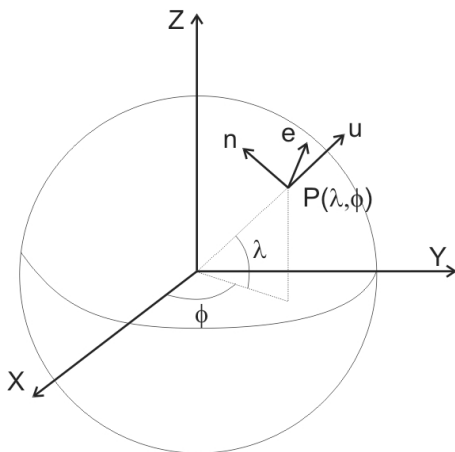
$$N = \frac{a^2}{\sqrt{a^2\cos^2\lambda + b^2\sin^2(\lambda)}}$$

$$b = -fa + a$$

- a : semi-major axis of ellipsoid (WGS84: 6378137.0 m)
- f : flattening of ellipsoid (WGS84: 1/298.257223563)
- b : semi-minor axis of ellipsoid
- λ, ϕ, h : geodetic latitude, longitude, height (above ellipsoid)
- X, Y, Z : ECEF Cartesian coordinates

Recall: Geocentric to Local

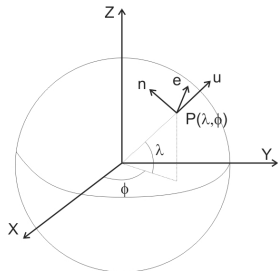
- origin of this datum is point of your choice on the surface of the Earth
- right handed coordinate system:
 - U is vertical. i.e., perpendicular to local equipotential surface, points up
 - N is in local horizontal plane and points to geographic north
 - E is in local horizontal plane and points to geographic east
- as ECEF (XYZ) units are meters, local units are meters, too



Recall: Geocentric to Local

Combine 3 rotations to align geocentric with NEU frame:

$$\begin{bmatrix} v_N \\ v_E \\ v_U \end{bmatrix} = \begin{bmatrix} -\sin(\lambda)\cos(\phi) & -\sin(\lambda)\sin(\phi) & \cos(\lambda) \\ -\sin\phi & \cos(\phi) & 0 \\ \cos(\lambda)\cos(\phi) & \cos(\lambda)\sin(\phi) & \sin(\lambda) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$
$$\vec{V}_{local} = R\vec{V}_{ecf}$$



Recall: Geocentric to Local

Combine 3 rotations to align geocentric with NEU frame:

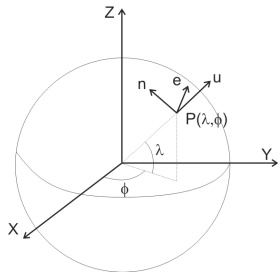
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$$\vec{V}_{local} = R \vec{V}_{ecef}$$

- inverse matrix of R can be used to convert local to ECEF
- since R is rotation matrix:

$$R^{-1} = R^T$$

- Therefore:

$$\vec{V}_{ecef} = R^T \vec{V}_{local}$$



Estimating Plate Angular Velocity

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} 0 & Z & -Y \\ -Z & 0 & X \\ Y & -X & 0 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Estimating Plate Angular Velocity

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} 0 & Z & -Y \\ -Z & 0 & X \\ Y & -X & 0 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$
$$d = Gm$$

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- ...
- Invert for m :

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = (G^T W G)^{-1} G^T W \vec{v}$$

Estimating Plate Angular Velocity

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- ...
- Invert for m :

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = (G^T W G)^{-1} G^T W \vec{v}$$

- where the weight matrix W is the inverse of the velocity covariance matrix C_V (comes from processing)
- model covariance matrix is $C_\omega = (G^T C_V^{-1} G)^{-1}$

Estimating Plate Angular Velocity

- How many site velocities do you need?

Estimating Plate Angular Velocity

- How many site velocities do you need?
- 3 parameters in plate angular velocity vector

Estimating Plate Angular Velocity

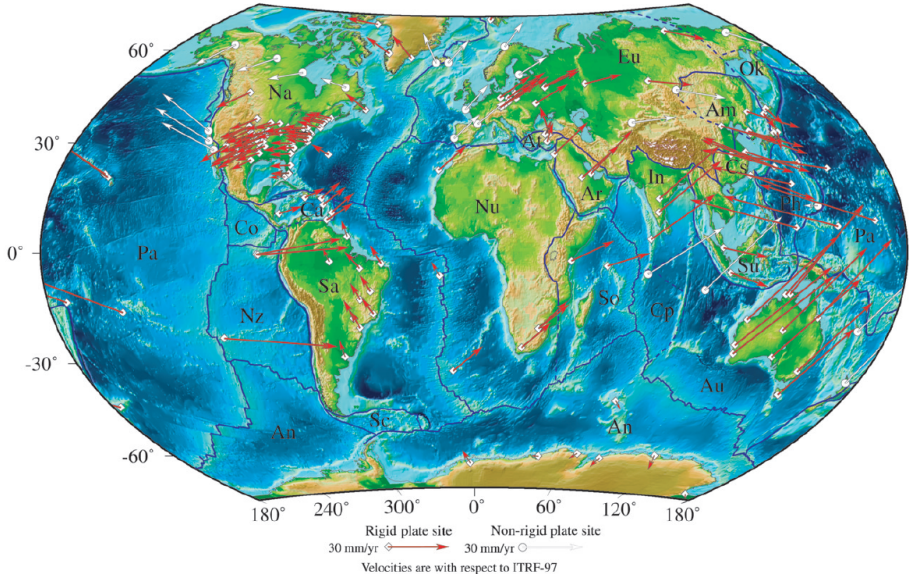
- How many site velocities do you need?
- 3 parameters in plate angular velocity vector
- 3 data in each site velocity (v_N , v_E , v_U)
- But: plate model predicts no vertical - only horizontals count!

Estimating Plate Angular Velocity

- How many site velocities do you need?
- 3 parameters in plate angular velocity vector
- 3 data in each site velocity (v_N , v_E , v_U)
- But: plate model predicts no vertical - only horizontals count!
- Need velocities for at least 2 sites to constrain plate angular velocity
- The more GPS velocities and the farther apart, the better determined is plate angular velocity

Example: REVEL-2000

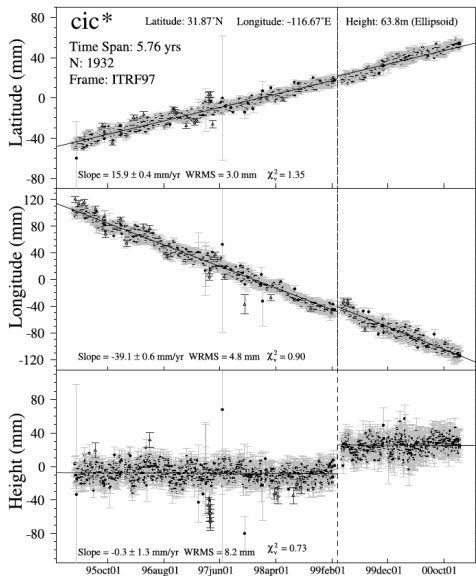
REVEL-2000



Example: REVEL-2000

- REVEL = “recent velocities”
- Global plate motion model based entirely on GPS data for 19 plates
- Data from 1993 through 2000.
- combination of many continuous sites and repeat campaign survey data
- first model with essentially global coverage
- 2/3 of tested plate pairs agree within uncertainties with NUVEL-1A (geologic 3Myr average)

Example: REVEL-2000



- example of data used in model
- long time series in ITRF97
- PPP solutions
- fit linear trends plus offset (here: combination of co-located sites)
- outlier rejection, quality control
- 345,000 station days

Example: GEODVEL



Argus et al., 2010, GJI

Rotation poles and confidence ellipses for adjacent plate pairs for GEODVEL (open circles, yellow) and NUVEL-1A (open squares, violet)

Example: GEODVEL

- GEODVEL = “GEODesy VELocity”
- based on GPS, VLBI, SLR, DORIS in ITRF2005
- relative angular velocities for 11 major plates
- also provides absolute plate poles

Example: GEODVEL



Example: GEODVEL

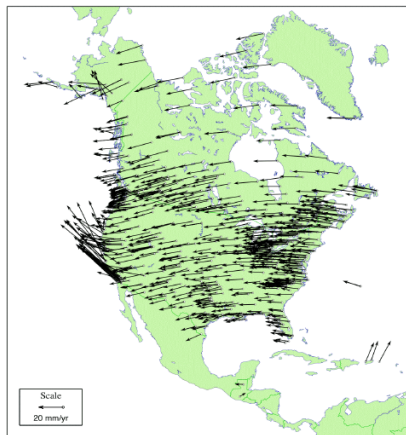


- "Velocities with respect to PLATE-NAME"
- very convenient for visualization purposes and modeling of tectonic deformation
- To convert into plate-fixed frame we need plate motion and velocities in the same geodetic frame (e.g., ITRF2008)
- Transformation:

Plate Fixed Reference Frames

- "Velocities with respect to PLATE-NAME"
- very convenient for visualization purposes and modeling of tectonic deformation
- To convert into plate-fixed frame we need plate motion and velocities in the same geodetic frame (e.g., ITRF2008)
- Transformation: subtract predicted motion based on plate angular velocity from observed velocity

Reference Frames – ITRF vs. fixed (stable North America)



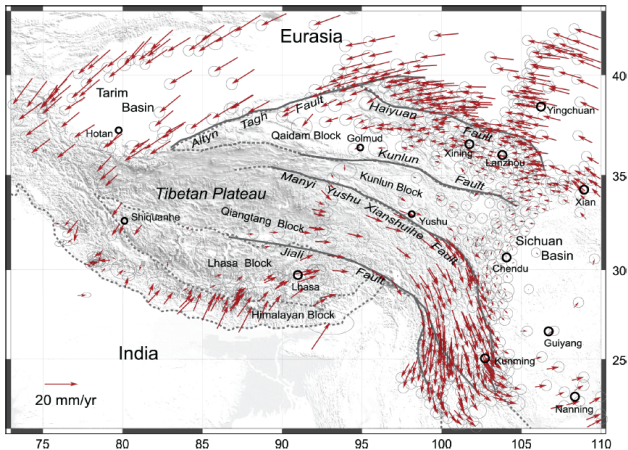
courtesy: Jeff Freymueller, UAF

Reference Frames – stable North America



- extension across Basin and Range
- Shear on San Andreas System
- Subduction strain in Cascadia, Alaska
- et al.

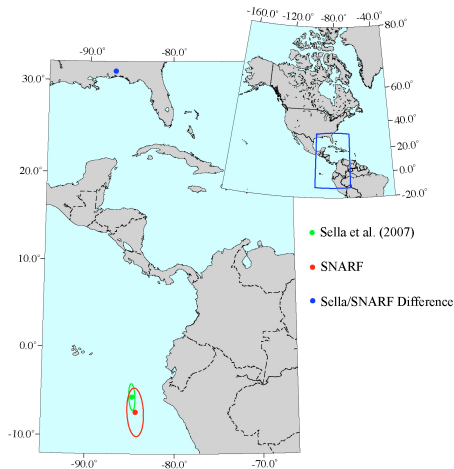
“Tibetan Plateau Reference Frame”



Gan et al. (2007) explained these motions in terms of a series of blocks separated by mostly strike-slip faults → plateau is deforming, but not changing area.

courtesy: Jeff Freymueller, UAF

NOAM Poles

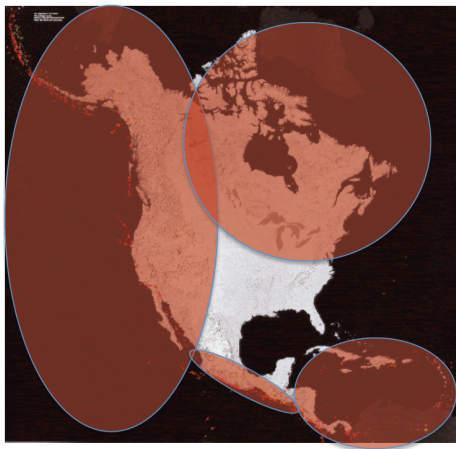


courtesy: Jeff Freymueller, UAF

- past studies: common that NOAM poles not within each others' confidence ellipses
- Difference between SNARF and Sella et al. (2007) is rotation about pole in SE US.

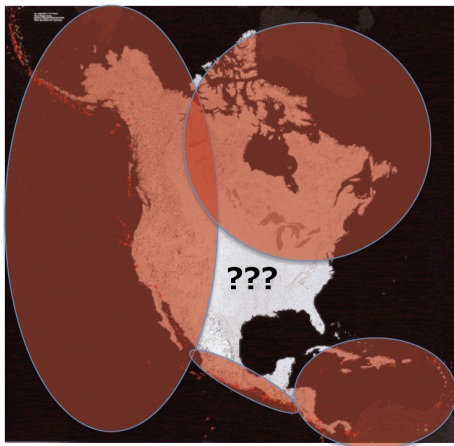
Why is NOAM Pole poorly determined?

Why is NOAM Pole poorly determined?



courtesy: Jeff Freymueller, UAF

Why is NOAM Pole poorly determined?



courtesy: Jeff Freymueller, UAF

- tectonics in western North America
- glacial isostatic adjustment in northern North America
- SE is thought to be stable on geologic and geodetic time scales
- limited area to determine plate angular velocity, susceptible to bias