



**ERTH 491-01 / GEOP 572-02**  
**Geodetic Methods**

**– Lecture 26: Modeling - Slip Inversion cont'd –**

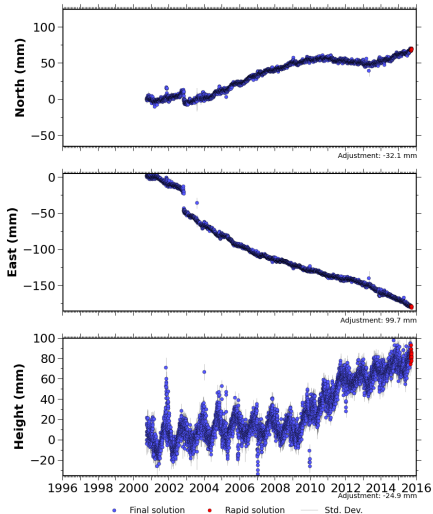
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November 16, 2015

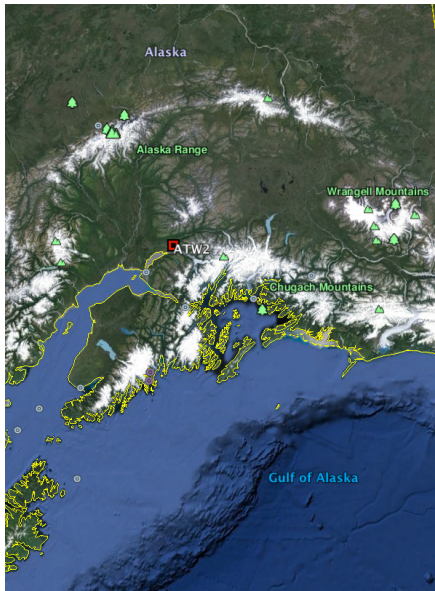
# Guess The $\approx 8$ Processes

## ATW2 (ATW2\_AKDA\_AK2000) NAM08

Processed Daily Position Time Series - Cleaned (Outliers Removed)



Source file: ATW2.pbo.nam08.pos Last epoch plotted: 2015-09-27 12:00:00



# Solving for Slip

data vector  
(observed  
displacements)

$$\begin{bmatrix} u_{e1} \\ u_{e2} \\ \dots \\ u_{n1} \\ u_{n2} \\ \dots \\ u_{u1} \\ u_{u2} \\ \dots \end{bmatrix} = \begin{bmatrix} \text{strike-slip} & \text{dip-slip} \\ g_1^{ss} & g_1^{ds} & \dots & g_N^{ds} \\ g_2^{ss} & g_2^{ds} & \dots & g_2^{ds} \\ \dots & \dots & \dots & \dots \\ g_N^{ss} & g_N^{ds} & \dots & g_N^{ds} \end{bmatrix} \begin{bmatrix} S_{ss,1} \\ S_{ss,2} \\ \dots \\ S_{ss,N} \\ S_{ds,1} \\ S_{ds,2} \\ \dots \\ S_{ds,N} \end{bmatrix}$$

patch 1      patch N

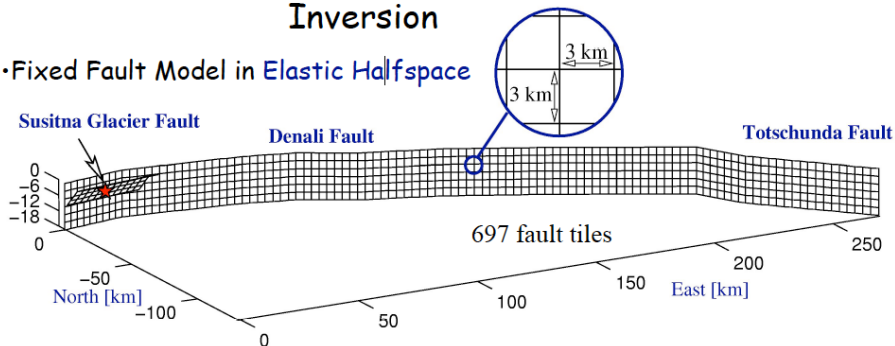
Vector of  
estimates  
(slip on each  
patch)

Eric Calais

# Solving for Slip

## Inversion

- Fixed Fault Model in Elastic Halfspace



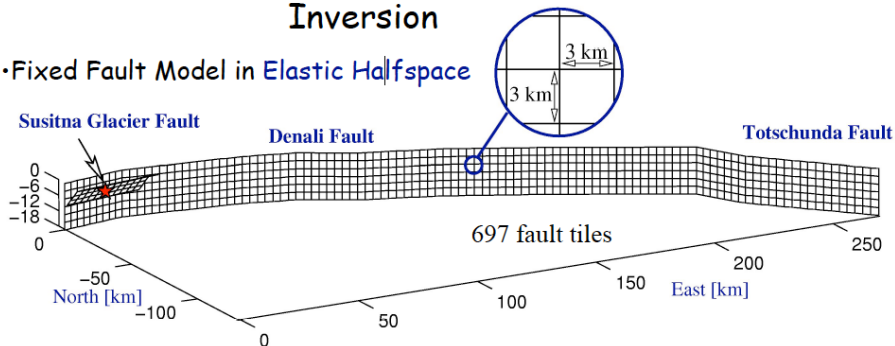
*Sigrun Hreinsdottir*

With 224 GPS sites and 697 fault tiles solving for dip-slip and strike-slip, what problem are we running into?

# Solving for Slip

## Inversion

- Fixed Fault Model in Elastic Halfspace



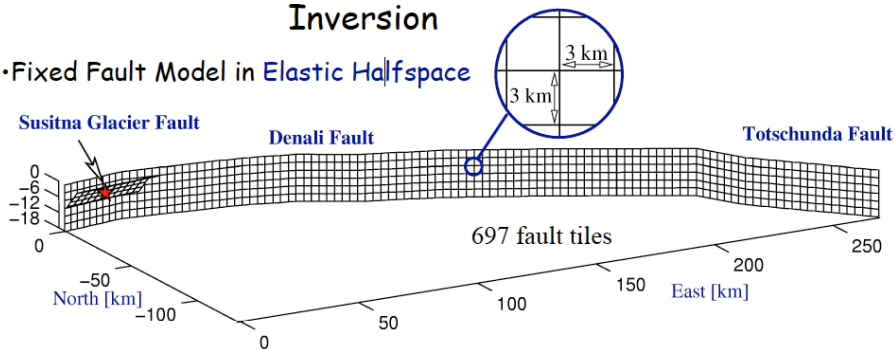
*Sigrun Hreinsdottir*

With 224 GPS sites and 697 fault tiles solving for dip-slip and strike-slip, what problem are we running into?  
Underdetermined system.

# Solving for Slip

## Inversion

- Fixed Fault Model in Elastic Halfspace



*Sigrun Hreinsdottir*

- observations at 225 GPS sites: 675 data (if vertical helps)
- 697 fault tiles, ss, ds: 1394 unknowns
- no enough data to constrain number of unknowns
- also often an issue: unphysical oscillatory slip

# Regularization / Smoothing

- Idea: Minimize the rate of change of slip with position
- “rate of change of slip” is curvature
- Laplacian:

$$\nabla^2 = \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2}$$

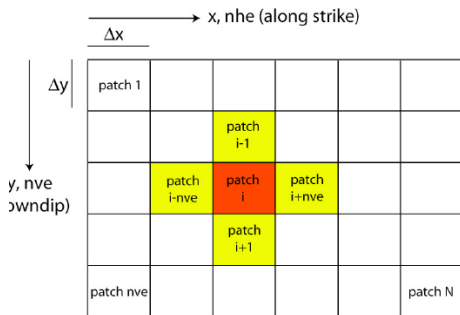
- Practice: Minimize sum of partial second differentials of slip for each fault patch
- Can be solved using finite-difference method for a function P

$$\frac{\delta^2 P(x)}{\delta x^2} \approx \frac{P(x - \Delta x) - 2P(x) + P(x + \Delta x)}{\Delta x^2}$$

# Regularization / Smoothing

- Our function  $P(x)$  is slip  $s$  which varies along-strike ( $x$ ) and down-dip ( $y$ )
- For patch  $i$  finite difference approximation of Laplacian is ( $nve$  = number of vertical elements,  $nhe$  = horizontal):

$$l_i = \frac{s_{i-nve} - 2s_i + s_{i+nve}}{\Delta x^2} + \frac{s_{i-1} - 2s_i + s_{i+1}}{\Delta y^2}$$



Eric Calais



# Regularization / Smoothing

- In practice, equation:

$$l_i = \frac{s_{i-nve} - 2s_i + s_{i+nve}}{\Delta x^2} + \frac{s_{i-1} - 2s_i + s_{i+1}}{\Delta y^2}$$

is written in matrix form, for the along-strike and down-sip components:

$$\begin{bmatrix} \dots \\ 0 \\ \dots \end{bmatrix} = \underbrace{\begin{bmatrix} \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ (0) & \frac{1}{\Delta x^2} & (0) & -\frac{2}{\Delta x^2} & (0) & -\frac{1}{\Delta x^2} & (0) \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}}_{L_x} \begin{bmatrix} \dots \\ s_{i-nve} \\ \dots \\ s_i \\ \dots \\ s_{i+nve} \\ \dots \end{bmatrix} \qquad \begin{bmatrix} \dots \\ 0 \\ \dots \end{bmatrix} = \underbrace{\begin{bmatrix} \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ (0) & \frac{1}{\Delta y^2} & (0) & -\frac{2}{\Delta y^2} & (0) & -\frac{1}{\Delta y^2} & (0) \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}}_{L_y} \begin{bmatrix} \dots \\ s_{i-1} \\ \dots \\ s_i \\ \dots \\ s_{i+1} \\ \dots \end{bmatrix}$$

- The 2 Laplacian matrices are then added:

$$L = L_x + L_y$$

# Regularization / Smoothing

- The original problem was:

$$[u] = [ G_{ss} \quad G_{ds} ] \begin{bmatrix} s_{ss} \\ s_{ds} \end{bmatrix}$$

- now it becomes:

$$\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} G_{ss} & G_{ds} \\ L & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} s_{ss} \\ s_{ds} \end{bmatrix}$$

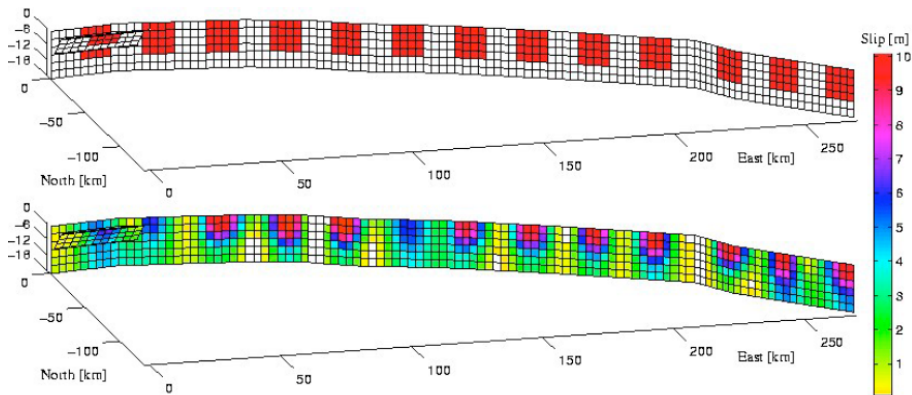
- amount of smoothing can be tuned using scalar smoothing factor  $\kappa$ :

$$\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} G_{ss} & G_{ds} \\ \kappa L & 0 \\ 0 & \kappa L \end{bmatrix} \begin{bmatrix} s_{ss} \\ s_{ds} \end{bmatrix}$$

- $\kappa = 0$ : no smoothing,  $\kappa = 1$  maximum smoothing

# Regularization / Smoothing

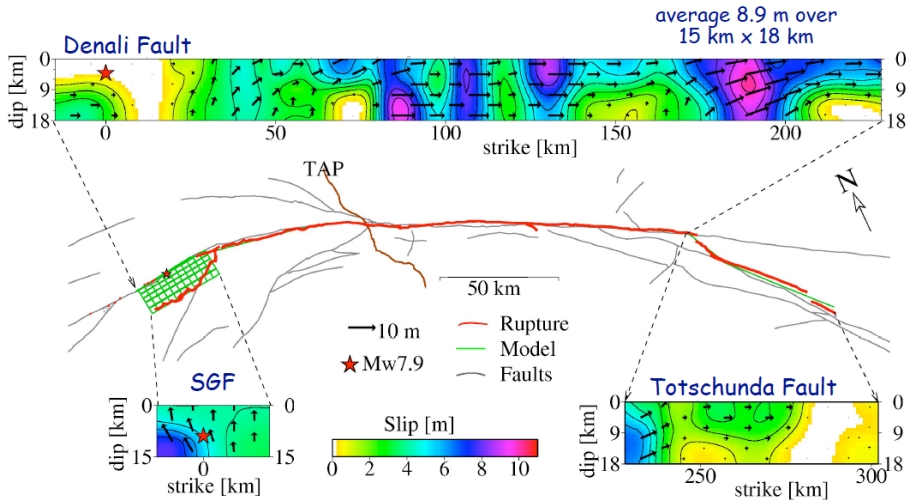
What can you recover? Checker board / Resolution test:



*Sigrun Heinsdottir*

# Distributed Slip Inversion

This is how you get this:



$M_0 = 6.81 \times 10^{20} \text{ Nm}$   
 $M_w 7.89$

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